Last Name: ______________________________

First Name: ______________________________

MATH 23b, SPRING 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Midterm (in-class portion)
March 18, 2002

Directions: You have one hour for this exam. No calculators, notes, books, etc. are allowed. Please answer on the pages provided. Show all work!

Problem	Points	Score
1	10	
2	5	
3	5	
4	10	
5	10	
6	10	
Total	50	

1. True or False

T	or	**F**	If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is differentiable at $\mathbf{a} \in \mathbb{R}^n$, then all of its directional derivatives exist at $\mathbf{a} \in \mathbb{R}^n$.
T	or	**F**	On the set of points $S = \{(x, y) \in \mathbb{R}^2 \| f(x, y) = 0\}$, where $f(x, y) = x^2 - y^2 - 1$, there is a neighborhood of the point $(1, 0)$ on which we may write $y = h(x)$ with $f(x, h(x)) = 0$.
T	or	**F**	If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is differentiable at $\mathbf{a} \in \mathbb{R}^n$, then $[D_i D_j f](\mathbf{a}) = [D_j D_i f](\mathbf{a})$, for all i and j.
T	or	**F**	If $f : \mathbb{R}^m \longrightarrow \mathbb{R}^k$ and $g : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ are both differentiable, then $f \circ g : \mathbb{R}^n \longrightarrow \mathbb{R}^k$ is also differentiable, and if $\mathbf{a} \in \mathbb{R}^n$, then $J(f \circ g)(\mathbf{a}) = Jf(\mathbf{a}) \cdot Jg(\mathbf{a})$.
T	or	**F**	Let $A \in M_n(\mathbb{R})$. Then A is symmetric if and only if A has an orthonormal eigenbasis.

2. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$.

   Define what it means for f to be *differentiable* at $\mathbf{a} \in \mathbb{R}^n$.

3. Let $f : \mathbb{R}^4 \longrightarrow \mathbb{R}$ be given by $f(x, y, z, w) = xyz - \cos(yzw)$. At the point $\mathbf{a} = (1, 2, 3, 0)$, find the directional derivative $D_{\mathbf{h}} f(\mathbf{a})$, where $\mathbf{h} = (6, 1, -1, 1)$ is the direction vector.

4. Consider the function $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ given by:

$$F(x, y, z) = (xy - y + 1 - e^{yz}, y^2 z)$$

(a) Find the Jacobian, ∇F.

(b) On the set $S = \{\mathbf{x} \in \mathbb{R}^3 | F(\mathbf{x}) = \mathbf{0}\}$, consider the point $\mathbf{a} = (1, 1, 0)$. Is it possible to express any two of the variables as a function of the third on S near $\mathbf{a}$? Explain.

5. Use the Lagrange multiplier method to find the point(s) on the parabola $x^2 - 4y = 0$ closest to the point $(0, b)$, where $b > 0$.

6. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be given by $f(x, y) = x^3 - 12xy + 8y^3$.

(a) Compute the gradient, ∇f.

(b) Find all critical points of f. *(Hint: There are two.)*

(c) Find the quadratic form $q(x, y)$ corresponding to the second-order derivative of f at each of the critical points.

(d) Find the eigenvalues of the matrix associated to q for each critical point.

(e) Classify the critical point by considering the "definiteness" of q at each critical point.