MATH 23b, SPRING 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Midterm (take-home portion)
March 18, 2002

Directions: You have until noon on Saturday, March 23, to complete this exam, when it should be turned in to my mailbox in the Science Center. You may use your own class notes, your own homework assignments, and any published books as your only aids. You may not use any internet resources except for the course website and the posted homework solutions. You may not discuss the exam with anyone, and all questions should be directed only to the instructor. (In particular, please do not direct questions to the Math 23b CA's.) Please note that I will hold office hours on Friday afternoon, but only until 4 P. M.

There is partial credit, but only for intelligible work. Please write neatly, and please turn in clean copies of solutions, not random scribblings that may or may not have anything to do with a final answer. In fact, one point per problem will be awarded for *neatness only*, and one point will be awarded for *style only*. Make sure your name is prominently displayed on your work, and *please* staple your final pages together into one stack.

You may quote results from class and/or your notes with an appropriate reference, and you must cite anything you take from a published book. Otherwise, all work should be your own. You may assume that we are always considering functions whose domains and ranges are finite-dimensional Euclidean spaces.

1. Show that if $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ is continuously differentiable, then f is not one-to-one.

2. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$. Recall the Laplacian of f is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

We define a function as *harmonic* if $\nabla^2 f = 0$ and *subharmonic* if $\nabla^2 f > 0$.

Let $D = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1\}$ be the closed unit disk. Show that if $f : D \longrightarrow \mathbb{R}$ is subharmonic, then f does not have a maximum on the interior of D.