6. a. Identify the ring $3\mathbf{Z}$.

$3\mathbf{Z} = \{x \in Z \text{ s.t. } x = 3k \text{ for some } k \in \mathbf{Z}\}$.

b. Define $\mathbf{Z}/3\mathbf{Z}$ using the appropriate equivalence relation.

We write $\mathbf{Z}/3\mathbf{Z} = Z/$ where x y iff $x - y \in 3\mathbf{Z}$ for all $x, y \in Z$. Alternatively, we may write $\mathbf{Z}/3\mathbf{Z} = \{[x] \mid [x] = [y] \text{ iff } x - y \in 3\mathbf{Z} \text{ for any } x, y \in \mathbf{Z}\}$.

c. Identify all of the equivalence classes in $\mathbf{Z}/3\mathbf{Z}$.

The equivalence classes of $\mathbf{Z}/3\mathbf{Z}$ are $[0] = \{\ldots - 3, 0, 3, 6, \ldots\}$, $[1] = \{\ldots - 2, 1, 4, 7, \ldots\}$, and $[2] = \{\ldots - 1, 2, 5, 8, \ldots\}$.

d. Construct tables for addition and multiplication in $\mathbf{Z}/3\mathbf{Z}$ using the definitions $[a]+[b] = [a + b]$ and $[a] \cdot [b] = [a \cdot b]$.

The tables are as follows:

+	[0]	[1]	[2]
[0]	[0]	[1]	[2]
[1]	[1]	[2]	[0]
[2]	[2]	[0]	[1]

·	[0]	[1]	[2]
[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]
[2]	[0]	[2]	[1]

e. Is $\mathbf{Z}/3\mathbf{Z}$ a field?

Yes. Properties A1-A4, M1-M3, and D are inherited directly from the integers. The element [0] is the additive identity and [1] is the multiplicative identity as evidenced in the tables above. It remains to show that M4 holds for $\mathbf{Z}/3\mathbf{Z}$, a property that is not true for the integers. In order for M4 to be true, we require that all nonzero elements of $\mathbf{Z}/3\mathbf{Z}$ have multiplicative inverses. Fortunately there are only two nonzero elements of $\mathbf{Z}/3\mathbf{Z}$ so we can check this directly. We see that $[1] \cdot [1] = [1]$ and $[2] \cdot [2] = [1]$, so both [1] and [2] are their own multiplicative inverse. Having verified all of the properties of a field, we conclude that $\mathbf{Z}/3\mathbf{Z}$ is a field.