Problem Set 1, Part E

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Problem 8: *Show that additive inverses in fields are unique, i.e. given $a \in F$, there exists a unique element $b \in F$ such that $a + b = 0$.*

Proof. Suppose on the contrary that there are two elements $b, c \in F$ such that $a + b = 0$ and $a + c = 0$. Then

$$b = b + 0 = b + (a + c) = b + a + c = c + a + b = c + 0 = c,$$

so $b = c$ and we are done. $\qquad\square$