Math 23a, Fall 2003

Problem Set 2, Part A
Solutions written by Tseno Tselkov

Problem 2: *Using the uniqueness of additive inverses in a ring, prove that* $(-a)(-b) = ab$*.*

Proof. We shall show that $(-a)b$ is an additive inverse of both ab and $(-a)(-b)$, which would imply that $ab = (-a)(-b)$ since additive inverses are unique. But we have

$$ab + (-a)b = (a + (-a))b = 0b = 0,$$

and

$$(-a)(-b) + (-a)b = (-a)(-b + b) = (-a)0 = 0,$$

and we are done. □

Problem 3: *If* V *is a vector space and* $v \in V$*, show that* $(-1)v = -v$*.*

Proof. We shall show that both $(-1)v$ and $-v$ are inverses of v, which would imply that $(-1)v = -v$ since inverses are unique. But we have

$$v + (-1)v = (1 + (-1))v = 0v = 0,$$

and also

$$v + (-v) = 0,$$

so we are done. □