Solution for HW2, part A
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Problem 2

For a set $A \subset \mathbb{R}^n$, show that $\partial A = \overline{A} \backslash A^o$.

Solution

Recall ∂A is the set of $x \in \mathbb{R}^n$ st for all $B_\epsilon(x)$, both $B_\epsilon(x) \cap A$ and $B_\epsilon(x) \cap A^c$ are nonempty.

$x \in A^o$ iff there exists $B_\epsilon(x)$ st $B_\epsilon(x) \cap A^c = \emptyset$. That is, (stating the converse now) $B_\epsilon(x) \cap A^c \neq \emptyset$ for all $\epsilon > 0$ iff $x \notin A^o$.

$B_\epsilon(x) \cap A \neq \emptyset$ for all $\epsilon > 0 \Leftrightarrow$ there exists a sequence of points in A converging to $x \Leftrightarrow x \in \overline{A}$.

Hence $\partial A = \{x \in \mathbb{R}^n | x \in \overline{A} \text{ and } x \notin A^o\} = \overline{A} \backslash A^o$.

Note: another strategy is to check inclusion both ways. If you do this, be sure to check *both* ways!.