

Problem Set 2, Part B – Solutions

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Problem 4

$$A = Q \times Q = \{(x, y) \in \mathbb{R}^2 \mid x, y \in Q\}$$

$$B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$$

I will just write the answers without justification, since the justifications are pretty straightforward.

(a)

$$A^\circ = \emptyset$$

$$(A^c)^\circ = \emptyset$$

$$\bar{A} = \mathbb{R}^2$$

$$\overline{(A^c)} = \mathbb{R}^2$$

$$\bar{A} \cap \overline{(A^c)} = \mathbb{R}^2$$

(b)

$$B^\circ = B$$

$$(B^c)^\circ = \{(x, y) \mid x^2 + y^2 > 1\}$$

$$\bar{B} = \{(x, y) \mid x^2 + y^2 \leq 1\}$$

$$\overline{(B^c)} = \{(x, y) \mid x^2 + y^2 \geq 1\}$$

$$\bar{B} \cap \overline{(B^c)} = \{(x, y) \mid x^2 + y^2 = 1\}$$

(c)

$$A \cap B = \{(x, y) \in \mathbb{Q}^2 \mid x^2 + y^2 < 1\}$$

$$\overline{(A \cap B)} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$$

$$(A \cap B)^\circ = \emptyset$$

$$\partial(A \cap B) = \overline{(A \cap B)} \setminus (A \cap B)^\circ = \overline{A \cap B}$$

□