Problem Set 4, Part D – Solutions

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Problem 6 $f : \mathbb{R}^2 \to \mathbb{R}^2$ with $f(x,y) = (sin(x-y), cos(x+y))$. Denote by $f_1 : \mathbb{R}^2 \to \mathbb{R}$ the function $f_1(x,y) = sin(x-y)$ and by $f_2 : \mathbb{R}^2 \to \mathbb{R}$, the function $f_2(x,y) = cos(x+y)$. Now, we ca simply apply the theory explained in class to the components of f.

We need to find the plane tangent to the graph of f in $\mathbb{R}^4$, at the point $(\pi/4, \pi/4, 0, 0)$ so:

$$z_1 - 0 = f_1(\pi/4, \pi/4) + \nabla f(\pi/4, \pi/4) \cdot (x - \pi/4, y - \pi/4)^t =$$
$$= 0 + (cos(\pi/4 - \pi/4), cos(\pi/4 - \pi/4)) \cdot (x - \pi/4, y - \pi/4)^t = x - \pi/4 - y + \pi/4 = x - y$$

Similarly,

$$z_2 - 0 = f_2(\pi/4, \pi/4) + \nabla f(\pi/4, \pi/4) \cdot (x - \pi/4, y - \pi/4)^t = -x + \pi/4 - y + \pi/4 = -x - y + \pi/2$$

So the two equations defining the tange plane are:

$$z_1 = x - y$$

$$z_2 = -x - y + \pi/2$$

□

Problem 7 If f is differentiable at a, then there exists a linear map $L : \mathbb{R}^n \to \mathbb{R}$ such that:

$$\lim_{\|h\| \to 0} \frac{f(a+h) - f(a) - L(h)}{\|h\|} = 0$$

Therefore, we must have that $\lim_{\|h\|\to 0}(f(a+h) - f(a) - L(h)) = 0 \Rightarrow \lim_{\|h\|\to 0}(f(a+h) - f(a)) = \lim_{\|h\|\to 0} L(h)$.

Since L is linear, then it is continuous and we have that $\lim_{\|h\|\to 0} L(h) = 0 \Rightarrow \lim_{\|h\|\to 0}(f(a+h) - f(a)) = 0$ which implies that f is continuous at a.

□