2. Use the Spectral Theorem to diagonalize the quadratic form $q(x, y, z) = xy + yz + xz$ and to classify its "definite"-ness.

By the Spectral theorem, we know that given a real symmetric matrix A and a quadratic form $q(x) = x^T Ax$ we may diagonalize A so that the diagonal contains its eigenvalues $\lambda_1, \ldots, \lambda_n$. Then with respect to the corresponding eigenbasis, the quadratic form $q(x) = \lambda_1 x_1^2 + \cdots + \lambda_n x_n^2$.

It is clear that we can represent the quadratic form $q(x, y, z) = xy + yz + xz$ with the symmetric matrix

$$A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

We find the eigenvalues of this matrix by computing the characteristic polynomial $\det(A - \lambda I) = -\lambda(\lambda^2 - \frac{1}{4}) - \frac{1}{2}(-\frac{1}{2}\lambda - \frac{1}{4}) + \frac{1}{2}(\frac{1}{4} + \frac{1}{2}\lambda) = -\lambda^3 + \frac{3}{4}\lambda + \frac{1}{4}$. So it suffices to find the roots of the cubic equation $4\lambda^3 - 3\lambda - 1 = 0$. We see right away that $\lambda = 1$ is a root, so we divide by $(\lambda - 1)$ to get the factorization $4\lambda^3 - 3\lambda - 1 = (\lambda - 1)(4\lambda^2 + 4\lambda + 1) = (\lambda - 1)(2\lambda + 1)^2 = 0$. So the eigenvalues of A are 1, $\frac{-1}{2}$, and $\frac{-1}{2}$. Thus with respect to the eigenbasis x_1, x_2, x_3 if $x = a_1 x_1 + a_2 x_2 + a_3 x_3$, then $q(x) = a_1^2 - \frac{1}{2}a_2^2 - \frac{1}{2}a_3^2$. So it is clear that q is indefinite (for example: consider the basis vectors).