Name: ______________________________

MATH 23b, SPRING 2004
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Midterm (part 2)
April 30, 2004

Directions: You have twenty-five minutes for this exam. No calculators, notes, books, etc. are allowed.

1. (2 points each) For each statement, please circle either **T** for **True** or **F** for **False**.

T or **F** Let $A \subset \mathbb{R}^n$ be compact, and let $f : A \to \mathbb{R}$ be continuous. If f attains its maximum (on$A$) at $\mathbf{a}$, then $\nabla f(\mathbf{a}) = \mathbf{0}$.

T or **F** Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be given by
$f(x, y, z) = (x + y + z,\ xyz,\ \ln(xyz))$,
and let $\mathbf{a} = (1, 1, 1) \in \mathbb{R}^3$, so that $f(\mathbf{a}) = (3, 1, 0)$. Then there is a neighborhood of $\mathbf{a}$ on which f is invertible.

T or **F** Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be differentiable and locally invertible in a neighborhood of $\mathbf{a}$. Then f^{-1} is differentiable at $f(\mathbf{a})$.

T or **F** The quadratic form
$q(x, y, z) = x^2 + 2xy + y^2 + 2yz + z^2$
is positive-definite.

T or **F** If $A \in M_n(\mathbb{R})$ is orthogonally diagonalizable, then A is symmetric.

T or **F** If $A \in M_n(\mathbb{R})$ is symmetric, then A is orthogonally diagonalizable.

For the next five statements, let $A \subset \mathbb{R}^n$ be a closed rectangle, let $f : A \to \mathbb{R}$ be bounded, and let P be a partition of A.

T or **F** If P' is a refinement of P, then $L(f, P) \leq L(f, P')$.

T or **F** If f is integrable on A, then $L(f, P) \leq \int_A f$.

T or **F** $\sup \{L(f, P)\} \leq \inf \{U(f, P)\}$

T or **F** If f is integrable on A, then f is continuous on A.

T or **F** ∂A has measure zero.

For the next two statements, let $A \subset \mathbb{R}^n$ be a closed rectangle, and let $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ both be integrable.

T or **F** If $f(x) \geq g(x)$ for all $x \in A$, then $\int_A f \geq \int_A g$.

T or **F** If $f(x) > g(x)$ for all $x \in A$, then $\int_A f > \int_A g$.

For the last two statements, let $A = [a, b] \times [c, d] \subset \mathbb{R}^2$ be a closed rectangle, and let $f : A \to \mathbb{R}$ be bounded. For any fixed $x \in [a, b]$, define $g_x : [c, d] \to \mathbb{R}$ by $g_x(y) = f(x, y)$.

T or **F** If f is integrable on A, then g_x is integrable on $[c, d]$.

T or **F** If g_x is integrable on $[c, d]$ for each $x \in [a, b]$, then

$$\int_A f = \int_a^b \int_c^d g_x(y)\, dy\, dx.$$

2. (4 points) Circle the letter a, b, c, or d, that corresponds to the correct statement.

   Let $C = \{(\frac{1}{n}, \frac{1}{m}) \mid n, m \in \mathbb{N}\} \subset \mathbb{R}^2$.

   a. C has measure 0 and content 0.

   b. C has measure 0 and undefined content.

   c. C has undefined measure and content 0.

   d. C has positive content.