MATH 23a, FALL 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Final Exam (in-class portion)
January 22, 2003

1. True or False (28 points, 2 each)

True	If V is a vector space and $S \subset V$ is a set of vectors that spans V, then S contains a basis for V.
True	If U and V are vector spaces over the same field F, then $(U \oplus V)/V \cong U$.
True	If $A, B \in M_n(F)$, then $\det(AB) = \det(A) \cdot \det(B)$.
False	$\operatorname{sgn}((134)(25)) = +1$
True	The number of *odd* permutations of n elements is $n!/2$.
True	Every alternating multilinear form $f : V^n \longrightarrow F$ is skew-symmetric.
True	If $dim(V) = m$ and $f : V^n \longrightarrow F$ is an alternating form with $n > m$, then $f = 0$.
False	If $A, B \in M_n(F)$ and $\operatorname{Spec}(A) = \operatorname{Spec}(B)$, then there is some invertible $S \in M_n(F)$ such that $A = SBS^{-1}$.
True	If V is a normed vector space with norm $|| \cdot ||$, then the function $d(x, y) = ||x - y||$ defines a metric on V.
False	If $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is continuous and $S \subset \mathbb{R}^n$ is open, then $f(S)$ is open in $\mathbb{R}^m$.
True	If $A \subset \mathbb{R}^n$ is open, then $A^\circ = A$, where A° is the interior of A.
False	If $\{A_n\}_{n=1}^\infty$ is any collection of open sets in $\mathbb{R}^n$, then $\cap_{n=1}^\infty A_n$ is open.
True	$\mathbb{Z}$ is closed as a subset of $\mathbb{R}$.
False	Any three non-zero vectors in $\mathbb{R}^3$ may be turned into an orthonormal basis via the Gram-Schmidt Orthogonalization Process.