MATH 23a, FALL 2004
THEORETICAL LINEAR ALGEBRA AND MULTIVARIABLE CALCULUS
(Final Version) Homework Assignment # 4
Due: November 5, 2004

1. To prepare for next week, read Sections 1.1–1.3 and 6.1 of Schneider and Barker.

2. (*) Read Chapter 2 of Schneider and Barker.

3. (*) For more on direct sums, read Halmos, sections 18–19, and Curtis, section 28.

4. (*) For more on quotient spaces, read Halmos, sections 21–22, and Curtis, section 26.

5. (A) Let $C[a,b] = \{f : [a,b] \to \mathbb{R} \mid f \text{ is continuous}\}$. Show that $C[0,1] \cong C[a,b]$ by constructing an explicit (linear) bijection.

6. (*) Find the inverse of the linear map $L : (\mathbb{Z}/7\mathbb{Z})^3 \longrightarrow (\mathbb{Z}/7\mathbb{Z})^3$ given by $L(x,y,z) = (x+y+z, 2x+3y+4z, 3x+4y+6z)$.

7. (B) Let F be any field. Show that the linear operator $L : F^2 \longrightarrow F^2$ given by $L(x,y) = (ax+by, cx+dy)$ is invertible if and only if $ad-bc \neq 0$.

8. (C) Let V be a vector space. We say that the linear map $P : V \to V$ is a **projection** provided that $P^2 = P$, that is: $P(P(\mathbf{v})) = P(\mathbf{v}), \forall \mathbf{v} \in V$. Define two subspaces of V as follows:

$$V_0 = \{\mathbf{v} \in V \mid P(\mathbf{v}) = \mathbf{0}\}$$

$$V_1 = \{\mathbf{v} \in V \mid P(\mathbf{v}) = \mathbf{v}\}$$

   Show that $V = V_0 \oplus V_1$.

9. (*) Prove the vector space distributivity laws for a quotient space.

10. (D) Let $P_n(\mathbb{R})$ be the vector space of polynomials (with real coefficients) of degree less than or equal to n, and let $n \geq 3$. Find a basis for $P_n(\mathbb{R})/P_2(\mathbb{R})$.

11. (*) Suppose U is a finite-dimensional vector space and V is a subspace of U. Show that $dim(U/V) = dim(U) - dim(V)$.