

A

Theorem: For any $k \in \mathbb{N}$,

$$\sum_{i=1}^k (2i - 1) = k^2 \quad (1)$$

Proof: (Note that this theorem can be rephrased as “show that the sum of the first k odd integers is equal to k^2 .”)

Using the language in which we first introduced Mathematical Induction, we let

$$G = \left\{ k \in \mathbb{N} \mid \text{the statement } \sum_{i=1}^k (2i - 1) = k^2 \text{ is true} \right\}.$$

1. To show the base step, we show that $1 \in G$. Note that for $k = 1$, the statement says $2(1) - 1 = 1^2$, and so $1 \in G$.
2. To show the inductive step, we show that *if* $k \in G$, *then* $k + 1 \in G$. That is, to complete the induction, we show that *if* (the theorem holds for k), *then* (the theorem holds for $k + 1$).

Assume that the theorem holds for k . So suppose

$$\sum_{i=1}^k (2i - 1) = k^2. \quad (2)$$

Then

$$\sum_{i=1}^{k+1} (2i - 1) = k^2 + 2(k + 1) - 1 = k^2 + 2k + 1. \quad (3)$$

Equivalently,

$$\sum_{i=1}^{k+1} (2i - 1) = (k + 1)^2 \quad (4)$$

which is the theorem for $i = k + 1$, and the inductive step is complete.

Thus, we have shown that:

- $1 \in G$
- if $k \in G$, then $k + 1 \in G$

and hence, by Mathematical Induction, $G = \mathbb{N}$. In other words, the statement is true for all natural numbers.