# SOLUTION SET 1D

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**Proposition 1.** *Every element in a field $F$ has a unique additive inverse.*

*Proof.* Let $b$ and $b'$ be additive inverses of $a$ in $F$. Consider the sum $b + a + b'$. On the one hand, it is equal to $(b + a) + b' = 0 + b' = b'$. On the other hand, by associativity, it is equal to $b + (a + b') = b + 0 = b$. Hence, by transitivity of equality, $b = b'$, and so additive inverses in fields are unique. □

Most people did this proof by contradiction. They assumed $b \neq b'$, and then did some operations showing that $b = b'$, which was a contradiction. While this is fine, notice that you did not use the assumption that $b \neq b'$. It is generally advisable to avoid proofs by contradiction when possible.

Also, there was some confusion about the "injectivity of addition of $x$" that we mentioned in class. Lots of people mentioned injectivity of addition in their proofs, without mentioning that it is addition *of something*. Furthermore, we have to prove this for $F$, which is some general field. Luckily, that is easy, for if $y + x = z + x$, then adding the inverse of $x$ to both sides gives us $y = z$.

Finally, a few people made the mistake of assuming what they were trying to prove. They said, suppose that $a + b = 0$ and $a + b' = 0$. Then, $b = 0 - a$, $b' = 0 - a$ so they must be equal. Unfortunately, subtracting $a$ from 0 is formally adding its inverse to 0, and we are entertaining the possibility that inverses are not unique. This proof thus relies on the fact that they *are* unique.

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