Solution Set 2E

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2 (E) For a set $A \subset \mathbb{R}^n$, show that $\partial A = \overline{A} \setminus A^\circ$.

Solution: Our strategy will be to show that $\partial A \subset \overline{A} \setminus A^\circ$ and that $\overline{A} \setminus A^\circ \subset \partial A$. Take $x \in \partial A$. Then $\forall \epsilon > 0$, $B_\epsilon(x) \cap A^c \neq \emptyset$. Hence, $x \notin A^\circ$. Now let $\epsilon_i = \frac{1}{i}$. Since $\forall \epsilon > 0$, $B_\epsilon(x) \cap A \neq \emptyset$, we can define a sequence of $y_i \in A$ such that $y_i \in B_{\epsilon_i}(x)$. Clearly, the y_i converge to x, so $x \in \overline{A}$. Thus, $\partial A \subset \overline{A} \setminus A^\circ$.

Now consider $x \in \overline{A} \setminus A^\circ$. Since $x \notin A^\circ$, $\forall \epsilon > 0, B_\epsilon(x) \cap A^c \neq \emptyset$, as for any $x \in A^\circ, \exists$ ϵ s.t. $B_\epsilon(x) \subset A$. Finally, since $x \in \overline{A}$, there exists a sequence of $y_i \in A$ converging to x. In other words, given any $\epsilon > 0, \exists N$ s.t. $||\mathbf{x} - \mathbf{y_i}\ || < \epsilon\ \forall i > N$. Hence, $B_\epsilon(x) \cap A \neq \emptyset\ \forall \epsilon > 0$. Thus, $\overline{A} \setminus A^\circ \subset \partial A$ as well, and so combining our results, we have that $\partial A = \overline{A} \setminus A^\circ$.