Math 23b Solution: Problem A

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3A

claim: The closure of $GL_n(R)$ is all of $M_n(R)$

proof: To show that the closure of $GL_n(R)$ is all of $M_n(R)$, note that the closure of $GL_n(R)$ is contained in $M_n(R)$ by the fact that any limit point of $GL_n(R)$ will be an nxn matrix.

So we are only worried about whether $M_n(R)$ is contained in the closure of $GL_n(R)$. Namely, we have to show that **if** $A \in M_n(R)$ **then** A is a limit point of $GL_n(R)$. Now, if A happens to be in $GL_n(R)$, then our inclusion is satisfied by the definition of closure. So we are only worried, in fact, about the case when A is NOT in $GL_n(R)$, namely, when $det(A) = 0$. If this is the case, we must show that there is a sequence $\{A_i\}$ of matrices converging to A such that $A_i \in GL_n(R)$ for every i. Consider the sequence of matrices $\{A - \frac{1}{n}\}$. This sequence converges to A obviously. Also, this sequence of matrices is nonsingular for all but at most n terms, so deleting the finitely many singular terms will give us a new sequence converging to A that is entirely contained in $GL_n(R)$. This new sequence is our desired sequence. ♡

note: A few of you likely did not attend a section or office hour where the strategy of this problem was discussed. Since I am so late getting this back to you, you can resubmit for some more credit. You can write the solutions on the back of your graded sheet–none of you used much more than one side.