Math 23b problem set 5
part A solution.

b.) $D_{\vec v} f(\vec 0) = \lim_{h\to 0} \dfrac{f(\vec 0 + h\vec v) - f(\vec 0)}{h}$

$= \lim_{h\to 0} \dfrac{f(h\vec v)}{h} = \lim_{h\to 0} \dfrac{(hx)^2(hy)}{h((hx)^2+(hy)^2)}$

$= \lim_{h\to 0} \dfrac{x^2 y}{x^2+y^2} = f(\vec v)$, if $\vec v = (x,y)$

c.) since the operator D_v satisfies $D_{t\vec v} = t D_{\vec v}$ and $D_v f(\vec 0) = f(v)$, it follows that

$f(t\vec v) = D_{t\vec v} f(\vec 0) = t D_v f(\vec 0) = t f(\vec v)$

d.) g diff'able $\Rightarrow D_{\vec v} g(\vec 0)$ exists $\forall \vec v \in \mathbb{R}^n$.

So $D_{\vec v} g(\vec 0) = \lim_{h\to 0} \dfrac{g(\vec 0 + h\vec v) - g(\vec 0)}{h} = \lim_{h\to 0} \dfrac{g(hv)}{h}$

pt c) $\Rightarrow D_v g(\vec 0) = \lim_{h\to 0} \dfrac{h g(\vec v)}{h} = g(v)$

g diffable $\Rightarrow D_v g(\vec 0) = \nabla g(\vec 0)\cdot \vec v = g(\vec v)$ from part

$\nabla g(\vec 0)$ is a <u>vector</u>, any map of the form $\vec v \longmapsto \vec x \cdot \vec v$ is <u>linear</u>.

e.) Note: Many of you didn't prove or conclude anything here. To get full credit you needed to compute $\nabla g(\vec 0) = \vec 0$, and then conclude from parts b) and d) that $g(\vec v) = \nabla g(\vec 0)\cdot \vec v = \vec 0 \cdot \vec v = \vec 0$, a contradiction.