

SOLUTION SET 8B

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MATH 23B
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(5) Let $q(x, y, z) = 2x^2 + 5y^2 + 2z^2 + 2xz$ be a quadratic form. Show that q is positive definite by:

(a) Using Theorem 8.8

(b) diagonalizing the quadratic form

The symmetric matrix representing the quadratic form is seen to be

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

In the language of Theorem 8.8, we have $\Delta_1 = \det(2) = 2$, $\Delta_2 = \det \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix} = 10$, and $\Delta_3 = \det A = 15$. Since all of these are positive, the form is positive definite.

Alternately, the characteristic equation is $\det(A - \lambda I) = 0$ which gives us (after simplification) $(\lambda - 5)(\lambda - 3)(\lambda - 1) = 0$, whence the eigenvalues of A are 1, 3, and 5. Hence, in some basis, A looks like

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

and the form represented by it is positive definite.

All of you got this one right.