1. (a) i.

$$\begin{aligned}(w,z) \in (X \cap Y) \times Z &\iff w \in X \cap Y \\ &\iff w \in X \text{ and } w \in Y \\ &\iff (w,z) \in X \times Z \text{ and } (w,z) \in Y \times Z \\ &\iff (w,z) \in (X \times Z) \cap (Y \times Z)\end{aligned}$$

Hence, $(X \cap Y) \times Z = (X \times Z) \cap (Y \times Z)$.

ii.

$$\begin{aligned}x \in Z \setminus (Y \cup Z) &\iff x \in Z, x \notin (Y \cup Z) \\ &\iff x \in X, y \notin Y \text{ and } x \notin Z \\ &\iff x \in X, x \notin Y \text{ and } x \in X, x \notin Z \\ &\iff x \in X \setminus Y \text{ and } x \in X \setminus Z \\ &\iff x \in (X \setminus Y) \cap (X \setminus Z)\end{aligned}$$

Hence, $X \setminus (Y \cup Z) = (X \setminus Y) \cap (X \setminus Z)$.

iii. $x \in X \setminus (Y \cap Z) \iff x \in X, x \notin (Y \cap Z)$. Since $x \notin (Y \cap Z)$, we know that $x \notin Y$, which holds iff $x \in X \setminus Y$, or $x \notin Z$, which is true iff $x \in X \setminus Z$. So x is either in $X \setminus Y$ or x is in $X \setminus Z$. Hence, $x \in (X \setminus Y) \cup (X \setminus Z)$, which implies that $X \setminus (Y \cap Z) = (X \setminus Y) \cup (X \setminus Z)$.

(b) i.

$$\begin{aligned}\alpha \in f^{-1}(Y \cap Z) &\iff f(\alpha) \in Y \cap Z \\ &\iff f(\alpha) \in Y \text{ and } f(\alpha) \in Z \\ &\iff \alpha \in f^{-1}(Y) \text{ and } \alpha \in f^{-1}(z) \\ &\iff \alpha \in f^{-1}(Y) \cap f^{-1}(Z)\end{aligned}$$

Hence, $f^{-1}(Y \cap Z) = f^{-1}(Y) \cap f^{-1}(Z)$.

ii.

$$\begin{aligned}b \in f^{-1}(Y \setminus Z) &\iff f(b) \in Y \setminus Z \\ &\iff f(b) \in Y, f(b) \notin Z \\ &\iff b \in f^{-1}(Y), b \notin f^{-1}(Z) \\ &\iff b \in f^{-1}(Y) \setminus f^{-1}(Z)\end{aligned}$$

And so $f^{-1}(Y \setminus Z) = f^{-1}(Y) \setminus f^{-1}(Z)$.

(c) i. The statement is true. Take x in $Y \cup Z$ and observe that $f(x) = f(Y \cup Z)$ means that, if $x \in Y$, then $f(x) \in f(Y)$, or if $x \in Z$, then $f(x) \in f(Z)$. Thus, $f(x) \in f(Y \cup Z) \iff f(x) \in f(Y) \cup f(Z)$, which means that $f(Y \cup Z) = f(Y) \cup f(Z)$.

ii. The statement is false. Counterexample via Hilary Havens: Take $f(x) = x^2$ and let $Y = \{-1\}, Z = \{1\}$. Then $Y \cap Z = \emptyset$ and $f(Y \cap Z) = \emptyset$, but $f(Y) \cap f(Z) = 1$, which is a contradiction. Hence $f(Y \cap Z) \neq f(Y) \cap f(Z)$.

(d) i. We'll first show that given a function $f \in (A^B)^C$, we can find a corresponding $h \in A^{B\times C}$. So take $f \in (A^B)^C, c_i \in C, b_j \in B, a_k \in A$. By definition, $f : C \longrightarrow A^B$, and so $f : c_i \longrightarrow g_i, g_i \in G$ the set of all maps g that send B to A. Thus, $g_i(b_j) = a_k$, and given (c_i, b_j), we can get an $a_k \in A$, which means that the desired function h is one such that $h(c_i, b_j) = a_k$, for $h \in A^{B\times C}$. Now, going in the other direction, given an $h \in A^{B\times C}$, we again take $c_i \in C, b_j \in B, a_k \in A$ such that $h(c_i, b_j) = a_k$. Given the set of all maps g from B to A, we know there exists one such g_i with $g_i(b_j) = a_k$. We then create a map f such that $f(c_i) = g_i$, and we have $f \in (A^B)^C$.

Therefore we have a bijection between $(A^B)^C$ and $A^{B\times C}$.