Math 25a – Honors Advanced Calculus and Linear Algebra
Problem Set 3, due Friday, October 31.

1. CS, p.210 #3
2. CS, p.115 #7 (Hint: you can either imitate the proof of the Rank-Nullity theorem, or apply the RNT to a suitable linear transformation $V_1 \times V_2 \to W$.)
3. CS, p.72-74 #13, #16-18, #29
4. CS, p.138 #16
5. CS, p.84 #10
6. Define a linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2, -x_2 - x_3, x_1 + 2x_2 + x_3, x_3 - x_1).$$

   Compute bases for $\mathrm{Im}(T)$ and $\mathcal{N}(T)$ (The Rank-Nullity theorem may be useful here).

7. Let V be a subspace of $\mathbb{R}^k$ (use any of the three norms we have discussed). Show that V is a closed subset of $\mathbb{R}^k$. (Hint: show that there is a number n and a linear transformation $T\colon \mathbb{R}^k \to \mathbb{R}^n$ so that V is the nullspace of T).
8. We have already seen the norm $\| \;\|_1$ on the vector space $\mathcal{C}([0,1])$ of continuous functions on $[0,1]$, defined by $\|f\|_1 = \int_0^1 |f(x)|dx$. We can define another norm $\| \;\|_\infty$ by $\|f\| = \max_{x\in[0,1]} |f(x)|$.

   Show that the linear transformation $T\colon \mathcal{C}([0,1]) \to \mathbb{R}^1$ given by $T(f) = f(0)$ is continuous for the norm $\| \;\|_\infty$, but not for $\| \;\|_1$. Thus these two norms are not equivalent.