Math 25a – Honors Advanced Calculus and Linear Algebra
Problem Set 4, due Friday, November 7.

Reminder: Midterm examination is Wednesday, November 12 in class.

1. CS, p. 84 #10

2. CS, p. 199 #3

3. Suppose $f\colon U \to \mathbb{R}$ is a function defined on an open set $U \subset \mathbb{R}^k$. We say that f has a local maximum at $x_0 \in U$ if there is an open set $U' \subset U$ so that $f(x) \leq f(x_0)$ for all $x \in U'$. The notion of local minimum is defined similarly.

   Now suppose that f is differentiable at $x_0 \in U$. Show that if the derivative $D_{x_0}f : \mathbb{R}^k \to \mathbb{R}$ is not the zero transformation, then f cannot have a local maximum or minimum at x_0.

4. Let $f\colon (a,b) \to \mathbb{R}$ be a differentiable function such that the derivative f' is continuous (we say f is in $\mathcal{C}^1((a,b))$). Let $[c,d]$ be a closed interval contained in (a,b). Show that there exists an $M > 0$ so that for all $x, y \in [c,d]$, $|f(x) - f(y)| \leq M|x - y|$. We say that f satisfies a *Lipschitz condition.* (Hint: use the Mean Value Theorem).