

Math 25b — Problem Set 6, due Friday, March 20.

1. CS, p. 352 #1, #7a, #14 (also look at #15)
2. CS, p. 357 #1, #3, #6, #12 (note the typo in #3 — it should read “ $f(x)g(x)|_a^b$ ”)
3. CS, p. 371 #10, #11, #12abcg, p. 379 #5 and p. 383 #6
4. (a) Suppose  $f: S \rightarrow \mathbb{R}$  is a nonnegative bounded function, and there is a set  $X \subset S$  of content zero with  $f(x) = 0$  if  $x \notin X$ . Show that  $f$  is integrable, and

$$\int_S f = 0.$$

(You can appeal directly to the definition of the integral, or else show that the closure  $\overline{X}$  also has content zero, and that  $f$  is continuous at all points  $x \notin \overline{X}$ )

- (b) Show that you can conclude the same thing if you don't assume  $f$  nonnegative.
- (c) Now suppose  $f$  and  $g$  are two bounded functions on  $S$  which are equal except on  $X$ . Show that  $f$  is integrable if and only if  $g$  is, and

$$\int_S f = \int_S g.$$