MATH 25A – PROBLEM SET #7
FRIDAY DECEMBER 3

1. Part A

1. Problems 2.7.3, 2.7.4(a) in the book. Example 2.7.1 shows that Newton's method for computing a square root reduces to a "divide and average" algorithm. In this problem you are asked to find a similar algorithm for computing k'th roots.
2. Problem 2.7.5 (a), (b) in the book + do two steps in Newton's method, not one.

2. Part B.

1. Problem 2.9.4 in the book.
2. To find a local inverse of a function $f(x)$ at a_0, we use Newton's method to solve $f(x) = y$ for x, with y fixed, and with the same initial guess a_0. This can be interpreted as follows. Define functions $a_i(y)$ assigning to each y the i'th guess in the Newton's method. The inverse to f will then be

$$f^{-1}(y) = \lim_{i \to \infty} a_i(y).$$

   (a) Write a formula for computing $a_{i+1}(y)$ from $a_i(y)$.
   (b) Use your formula to compute a_1, a_2 and a_3 for the function $f(x) = \ln(x)$ at $a_0 = 1$. Compare $a_3(.1)$ with the correct value of $f^{-1}(.1)$.

3. Part C.

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be $f(x, y) = y^2 - x^2(x + 1)$, and let X be the plane curve

$$C = \{(x, y) | f(x, y) = 0\}.$$

   (a) Draw C.
   (b) Find all points $a \in C$ near which C is given explicitly by $y = g(x)$ or $x = g(y)$.
2. We proved in class that, given a C^2 function $f : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^n$, and a point $c \in \mathbb{R}^m \times \mathbb{R}^n$ such that $f(c) = 0$ and

$$\frac{\partial f}{\partial y}(c)$$

   is an invertible matrix, then in a neighborhood of c, the set

$$W = \{(x, y) | f(x, y) = 0\}$$

   is explicitly given by

$$W = \{(x, y) | y = g(x)\}$$

   for some function g.
   Using this, prove Theorem 2.9.9 in the book, assuming that F is C^2.

4. Part D.

1. For the function f in Problem 2.9.4 in the book, find $L_N(f)$ and $U_N(f)$ for all N. What is the integral?
2. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous with bounded support. Without referring to Section 4.3 in the book, prove that f is integrable. (Hint: you have to show that $U_N - L_N$ approaches 0 as $N \to \infty$. But this difference is the sum of the areas of certain rectangles. To prove that this sum approaches zero, you may want to use the fact that f is uniformly continuous (which follows from f being continuous with bounded support)).