MATH 25A – PROBLEM SET #5
DUE FRIDAY NOVEMBER 1ST

1. Part A

(1) Problem 2.1.4 in the textbook.
(2) Problem 2.2.11 in the textbook (I want to see most of the computations written down).
(3) Problem 2.2.8 in the textbook (note that in the text of 2.2.6 (a) there's a typo: the word "unique" shouldn't be there).
(4) Problem 2.3.2 in the textbook.
(5) Problem 2.3.4 in the textbook.

2. Part B

(1) Problem 2.3.5 in the textbook.
(2) Problem 2.3.7 in the textbook.

For this part of the homework, please read about elementary matrices on pgs 163-165. The book shows that an elementary row operation of A can be given as a multiplication EA of A with an elementary matrix E. One can also define elementary column operations, which are given as multiplications AE of A with E from the right.

(3) A matrix $A \in Mat(n,n)$ is called *upper triangular* if $A_{ij} = 0$ for $i > j$, and *lower triangular* if $A_{ij} = 0$ for $i < j$. Prove that if A and B are upper triangular (resp. lower triangular), then AB is upper triangular (resp. lower triangular).
(4) A matrix $P \in Mat(n,n)$ is called a *permutation matrix* if the entries of P are zeroes and ones, and every row and every column of P contains exactly one one. Multiplying a matrix A with a permutation matrix from the left has the effect of permuting the rows of A. Prove that if A is any matrix then we can write

$$PA = LU$$

for some permutation matrix P, lower triangular matrix L, and upper triangular matrix U. (Hint: consider the row reduction in matrix form. First do the case where you don't have to exchange any rows.)

(5) Prove that for any matrix A there exist invertible matrices P and Q such that

$$PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

for some r. (Hint: use row and column reduction.)