MATH 25A – PROBLEM SET #8
DUE FRIDAY DECEMBER 6TH

1. Part A

(1) Prove the following version of the inverse function theorem:

Theorem. *A mapping $f : \mathbb{R}^n \to \mathbb{R}^n$ has a local inverse at a point a_0 if there exists a ball $B_R(a_0)$ such that:*
- *$Df(x)^{-1}$ exists and satisfies $|Df(x)^{-1}| \leq K$ for some constant K, for every $x \in B_R(a_0)$.*
- *$Df(x)$ is Lipschitz with constant M on $B_R(a_0)$.*
- *$|f(x) - f(a_0)| \leq \frac{1}{4MK^2}$ for all $x \in B_R(a_0)$.*

This theorem is effective when we try to estimate the radius of a ball on which the inverse function is defined. Note that an (apparently) stronger version is proved in the book, but I'd really like to see the proof of this particular statement – of course it has to do with Newton's method for the function $f_y(x) = f(x) - y$ defined for a fixed y.

(2) Problem 2.7.12 in the textbook.

2. Part B

(1) Problem 2.9.7 in the textbook.
(2) Problem 2.9.10 in the textbook. Replace the matrix given there by the identity matrix I_2. In other words: does every matrix close to the identity I_2 have a square root?
(3) Problem 2.9.11 in the textbook.
(4) Problem 2.32 in the review section for Chapter 2.