Math 25a Homework 4

Due Tuesday 18th October 2005.

Half of this problem set will be graded by Alison and half by Ivan. Please turn in problems from Section 1 separately from the problems in Section 2. Remember to staple each bundle of solutions and also to put your name on each!

For some of the questions in this week's HW, Rudin gives a lot of hints or suggestions as to how to proceed. If you use these, make sure you justify each step he gives!

1 Alison's problems

(1) Problem 17 on page 44 of Rudin.

(2) Problem 19 on page 44 of Rudin.
The definition of separated sets in on page 42 of Rudin.

(3) Problem 20 on page 44 of Rudin.

(4) Problem 23 on page 45 of Rudin.
The definition of separable is in problem 22 on page 45 of Rudin.

(5) Problem 25 on page 45 of Rudin.
The hint makes this problem straightforward to do, but it is an important result!

2 Ivan's problems

(1) Problem 26 on page 45 of Rudin.
You may assume the results of problems 23 and 24 on page 45 of Rudin.

(2) Problem 27 page 45 of Rudin.
Now read through problem 28. This result is an immediate corollary of problem 27. You do not need to hand in problem 28 - just read and appreciate it.

(3) Let K be a compact subset of the metric space (X, d_X) and let L be a compact subset of the metric space (Y, d_Y). The *product metric* on the set $X \times Y$ is

$$d((x_1, y_1), (x_2, y_2)) = d_X(x_1, x_2) + d_Y(y_1, y_2).$$

This is a metric on the set $X \times Y$ (you don't need to prove this). Show that $K \times L$ is a compact subset of $X \times Y$.

Optional: What does this problem tell you about k-cells (section 2.17 of Rudin) and $\mathbb{R}^k$*? To answer this, you'll have find a different looking product metric.*

(4) Problem 30 on page 46 of Rudin.

3 The Math Puzzler - just for fun!

Each week there will be a "math puzzler" for the class to think about. Please feel free to submit a "puzzler" you think the class might enjoy. The "puzzlers" don't have to be difficult, nor related to the material in class — they just have to be fun to think about!

This problem is taken from *Math Horizons* Sept 2005.

You enjoy doing origami (i.e. folding things like flowers or boxes out of paper) and are a purist. So you don't use scissors, rulers, etc — it is just you, the square sheet of paper and your brain. You have decided that in order to fold your latest and greatest new origami model you simply must be able to divide the side of a square sheet of paper into n equal pieces for any given positive integer value of n. How can this be accomplished without resorting to "unacceptable" methods? *(Hint: Can you divide the paper into thirds?)*