Name: ______________________________

MATH 134
Elizabeth Denne

Due: 10/15/2004

Worksheet 3

1. $O(n)$ = "the orthogonal group" = all choices of orthonormal basis $\{\overrightarrow{e_1}, \ldots, \overrightarrow{e_n}\}$ of $\mathbf{R^n}$ = all $n \times n$ real matrices A such that $AA^T = I$.

a) $O(4)$ is the solution set of ????? equations in the variables

$$(x^1, x^2, x^3, x^4, y^1, \ldots, z^1, \ldots, w^1, \ldots, w^4).$$

Write these equations. (Note: you can also write these variables as $(\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, \overrightarrow{e_4})$ where $\overrightarrow{e_i} \in \mathbf{R^4}$.)

b) Write down the ???×16 Jacobian matrix Dg:

c) Show that $Dg(A)$ has full rank (that is, its rows are linearly independent) for every $A \in O(4)$. So, by the Implicit Function Theorem, $O(4)$ is a manifold. (There is a really easy way, and a not-too-hard way.)

d) By Implicit Function Theorem, any $A \in O(4)$ is in a coordinate patch whose coordinates are chosen from $(x^1, \ldots, w^4)$. Give a choice that will work for $A = I$. Please explain.

2. The same argument works to show $O(n)$ is a manifold. Show that $O(n)$ is $n(n-1)/2$ dimensional. Justify in two ways:

a) By the definition of $O(n)$ above.

b) By the IFT.

We conclude that any point of $O(n)$ has a patch whose coordinates are given by projection onto one of the coordinate $n(n-1)/2$ planes of $\mathbf{R}^{n^2}$. (Note: different coordinate $n(n-1)/2$ planes for different points!) We say "$O(n)$ is an $n(n-1)/2$-dimensional submanifold of $\mathbf{R}^{n^2}$".