Name: _____

MATH 134
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Due: 10/22/2004

Worksheet 4

1. Let $M = U = \mathbf{R}^2 = \{(x^1, x^2)\}$ and set $y^1 = (x^1)^2 + (x^2)^2$ and $y^2 = x^1 x^2$.

a) For what point(s) (x^1, x^2) can you NOT use (y^1, y^2) as coordinate on some open set about (x^1, x^2)? Explain.

b) Let p_0 be the point with $(x^1, x^2)(p_0) = (1, \frac{1}{2})$. Express $\frac{\partial}{\partial y^1}|_{p_0}$ and $\frac{\partial}{\partial y^2}|_{p_0}$ as linear combinations of $\frac{\partial}{\partial x^1}|_{p_0}$ and $\frac{\partial}{\partial x^2}|_{p_0}$.

c) Sketch the curves $y^1 = \frac{1}{2}, 1, 2$ and $y^2 = 0, \pm\frac{1}{2}, \pm 1, \pm 2$. Please use a large diagram (at least half a page) to make both the drawing clear and the grading easier!!

d) Sketch the tangent vectors $\frac{\partial}{\partial y^1}|_{p_0}$ and $\frac{\partial}{\partial y^2}|_{p_0}$ from (b).

e) Explain why $\phi = (y^1, y^2)$ is NOT one-to-one on **any** open set around a point from (a).

2. Convince me by a sketch that the Klein Bottle has a never-zero vector field:

3. Find a never-zero vector field on any odd-dimensional sphere S^{2n-1}.
HINT 1: $S^{2n-1} = \{\vec{x} = (x^1, x^2, \ldots, x^{2n-1}, x^{2n}) \in \mathbf{R}^{2n} \mid ||\vec{x}|| = 1\}$. Then $(S^{2n-1})_{\vec{x}} = \{\vec{y} \in \mathbf{R}^{2n} \mid \vec{x} \cdot \vec{y} = 0\}$. For each $\vec{x} \in S^{2n-1}$, you must find such a $\vec{y} \neq \vec{0}$.
HINT 2: Try it for $S^1 \subset \mathbf{R}^2$ first.