MATH 134
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Due: 10/29/2004

Worksheet 5

1. Let $Y = x^2 \frac{\partial}{\partial x^1} + (x^2)^2 \frac{\partial}{\partial x^2}$, a vector field on $\mathbf{R}^2$.

a) Sketch Y and its flow lines (integral curves).

b) Find the flow line $\gamma(t)$ with $\gamma(0) = (a, b)$ (for $b \neq 0$, $b = 0$).

c) For what values of t is $\gamma(t)$ defined? (For $b > 0$, $b < 0$, $b = 0$.)

d) Find all points $p = (a, b)$ for which $\phi_1(p)$ exists. (That is, $\gamma_p(1)$ exists, where γ_p is the maximal flow line of Y with $\gamma_p(0) = p$.) Clearly mark these points p on a sketch.

2. As a particle moves along a flow line, its speed = magnitude of velocity vector = $|Y|$. Explain how the flow lines can be infinitely long in both directions, even though their time intervals are not. (Perhaps think about this in $\mathbf{R}^2$.)

3. The torus of revolution T (shown below) has a coordinate chart $(U, \phi = (u^1, u^2))$, $0 < u^1, u^2 < 2\pi$, and a map $F : T \to \mathbf{R}^3$, where F is given in coordinates by $x^1 = (4 + \cos(u^1)) \cos(u^2)$, $x^2 = (4 + \cos(u^1)) \sin(u^2)$, $x^3 = \sin(u^1)$.

a) Express $F_*(\frac{\partial}{\partial u^1})$ and $F_*(\frac{\partial}{\partial u^2})$ as linear combinations of $\frac{\partial}{\partial x^1}$, $\frac{\partial}{\partial x^1}$, $\frac{\partial}{\partial x^1}$.

b) Illustrate $F_*(\frac{\partial}{\partial u^1})$ and $F_*(\frac{\partial}{\partial u^2})$ carefully on the sketch, for several points of T. (Or you may attach your own sketch.)