Name: ______________________________

MATH 134
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Due: 11/12/2004

Worksheet 6

Please do at least 3 of these questions.

1. For $\vec{u}, \vec{v} \in \mathbf{R}^n$, set $W(\vec{u}, \vec{v}) = \vec{u} \cdot (\vec{u} + \vec{v})$. Is W a 2-covariant tensor? Explain.
2. Questions 1 and 2 from G&P Ch4 section 2 pg 160.
3. Question 3 from G&P Ch4 section 2 pg 160.
4. Question 4 from G&P Ch4 section 2 pg 160.
5. We regard dot product, $\cdot$, as a twice covariant tensor field g on $M = \mathbf{R}^3$. That is,

$$g(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w}.$$

Thus the components of g with respect to the standard coordinates (x^1, x^2, x^3) are:

$$g_{ij} = g(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}) = \frac{\partial}{\partial x^i} \cdot \frac{\partial}{\partial x^j} = \delta_{ij}.$$

Find the components of $g'_{ij} = g(\frac{\partial}{\partial x'^i}, \frac{\partial}{\partial x'^j})$ of g with respect to spherical coordinates $x'^1 = \rho$, $x'^2 = \theta$, $x'^3 = \phi$.

6. We regard cross product, $\times$, as a mixed tensor field W on $M = \mathbf{R^3}$, where W is 2-covariant and 1-contravariant, by:

$$W(\alpha, \vec{v}, \vec{w}) = \alpha(\vec{v} \times \vec{w}).$$

Thus the components of W with respect to the standard coordinates (x^1, x^2, x^3) on $\mathbf{R}^3$ are:

$$W^i_{jk} = W(dx^i, \frac{\partial}{\partial x^j}, \frac{\partial}{\partial x^k}) = dx^i(\frac{\partial}{\partial x^j} \times \frac{\partial}{\partial x^k}) =$$

Find the components of $W'^i_{jk} = W(dx'^i, \frac{\partial}{\partial x'^j}, \frac{\partial}{\partial x'^k})$ of W with respect to spherical coordinates $x'^1 = \rho$, $x'^2 = \theta$, $x'^3 = \phi$. (HINT: you know $W'^i_{jk} = \frac{\partial x'^i}{\partial x^l} \frac{\partial x^m}{\partial x'^j} \frac{\partial x^s}{\partial x'^k} W^l_{ms}$. But it is quicker to calculate W'^i_{jk} directly from the definition.)

Note that the 3-covariant tensor given by $W(\vec{u}, \vec{v}, \vec{w}) = \vec{u} \cdot \vec{v} \times \vec{w}$ has components W_{ijk} that are different from these. What is the relation between the W_{ijk} and the W'^i_{jk}? (HINT: this is meant to be a one sentence answer. Think about g_{ij} from q5.)

7. Question 11 from G&P Ch4 section 2 pg 161.