Math 25a – Honors Advanced Calculus and Linear Algebra
Problem Set 1, due Friday, October 17.

Homework policy: Homeworks will be accepted one class period after they are due, but with a 10% reduction in the grade. In addition, I will drop the lowest homework score in figuring your final grade. No homeworks will be accepted more than one lecture late. Homeworks are due at the **beginning** of class.

Any references to Corwin and Szczarba will be denoted "CS".

1. CS p. 38 #5

2. CS p. 45 #6-8

3. CS p. 101 #3, #13

4. Suppose W_1 and W_2 are subspaces of a vector space V. Show that $W_1 + W_2 = \{w_1 + w_2 \mid w_1 \in W_1,\ w_2 \in W_2\}$ is a subspace of V.

5. Let $T\colon V \to W$ be a linear transformation, and let S be a set of vectors which span V. Show that the set
$$T(S) = \{Ts \mid s \in S\}$$
is a spanning set for $\operatorname{Im} T$.

6. Find, with proof, a basis for $\operatorname{Ker} T$, where $T\colon \mathcal{P}_3 \to \mathbb{R}$ is the linear transformation
$$T(p) = p'(0) + p(1).$$

7. Let V and W be vector spaces. We can put a vector space structure on the set $V \times W$ of ordered pairs (v, w) with $v \in V$, $w \in W$ as follows:
$$(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2),$$
$$a(v, w) = (av, aw).$$
Convince yourself that this satisfies the axioms (you needn't write it up). This vector space is called the direct sum of V and W and is usually written $V \oplus W$.

   Let $T\colon V \to W$ be a linear transformation. Show that the *graph* of T, $G(T) = \{(v, T(v)) \mid v \in V\}$ is a subspace of $V \oplus W$.