Math 25a – Honors Advanced Calculus and Linear Algebra
Problem Set 6, due Wednesday, December 3.

1. CS, p. 120 #7, #8

2. CS, p. 129 #9, #19

3. Let V be a finite dimensional inner product space, and let V^* be the dual space.

   Given a vector $\vec{v} \in V$, define $\alpha_{\vec{v}} \in V^*$ by $\alpha_{\vec{v}}(\vec{w}) = \langle \vec{v}, \vec{w} \rangle$. Show that the function $T\colon V \to V^*$ given by $T(\vec{v}) = \alpha_{\vec{v}}$ is an injective linear transformation.

   If V is finite dimensional, conclude that T is an isomorphism. This gives another proof of the Riesz representation theorem.

4. Put an inner product on the space $\mathcal{P}_3$ of polynomials with degree ≤ 3 by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx.$$

   (Think about why this is an inner product). Apply the Gram-Schmidt procedure to the basis $\{1, x, x^2, x^3\}$, and then to the same basis in the opposite order.

5. Suppose we have a differentiable path $\alpha\colon \mathbb{R} \to \mathbb{R}^n$, and that $\|\alpha(t)\|_2 = 1$ for all $t \in \mathbb{R}$. Show that at any time t, the velocity vector $\alpha'(t)$ is perpendicular to $\alpha(t)$. (Can you see how this relates to CS p84, #10?)