MATH 25A – PROBLEM SET #9
FRIDAY DECEMBER 17

1. Part A

1. Cramer's Rule.
   (a) If $a_1, \ldots, a_n \in \mathbb{R}^n$, and $c_1, \ldots, c_n \in \mathbb{R}$, show that
   $$\det(a_1, \ldots, a_{i-1}, c_1 a_1 + \ldots + c_n a_n, a_{i+1}, \ldots, a_n) = c_i \det(a_1, \ldots, a_n).$$
   (b) Suppose we want to solve
   $$Ax = b$$
   where $A \in Mat(n,n)$ and $x, b \in \mathbb{R}^n$. Prove that if A is invertible then there is a unique solution x given by
   $$x_i = \frac{\det(a_1, \ldots, a_{i-1}, b, a_{i+1}, \ldots, a_n)}{\det(a_1, \ldots, a_n)},$$
   where a_i is the i'th column of A.

2. Part B.

1. Let P be a bounded convex polygon in $\mathbb{R}^2$ containing 0. (A polygon is a region bounded by line segments, for example a triangle, a parallelogram, a pentagon.)
   (a) Let the corners of P be $v_1, \ldots, v_n$. Find the area of the polygon by considering triangles with vertices $0, v_i, v_j$.
   (b) Is your formula true if the polygon does not contain 0 ? Is it true if the polygon is not convex?

3. Part C.

Problem 4.10.8 in the book.

4. Part D.

Problem 4.10.17 in the book