

Math 25a Homework 7

Due Tuesday 8th November 2005.

Half of this problem set will be graded by Alison and half by Ivan. Please turn in problems from Section 1 separately from the problems in Section 2. Remember to staple each bundle of solutions and also to put your name on each!

As we all have to suffer through a midterm exam next week, I've tried to make this week's HW a) shorter and b) easier than usual. Warning — the next set will be more challenging!

1 Alison's problems

- (1) Problem 5 on page 19 of Axler.
- (2) Problem 6 on page 19 of Axler.
- (3) Problem 9 on page 19 of Axler.

2 Ivan's problems

- (1) Problem 13 on page 19 of Axler.
- (2) Problem 15 on page 19 of Axler.
- (3) Problem 1 on page 35 of Axler.
- (4) Problem 2 on page 35 of Axler.

3 Warm up and Extra Problems.

There are HEAPS of great linear algebra problems. Problems 3 and 4 on page 19 of Axler get you to think about some more consequences of the vector space axioms. Problems 7, 8 and 10 on page 19 of Axler get you thinking about subspaces. Question 14 is a nice example to help get you used to thinking about the vector space of polynomial functions.

4 The Math Puzzler - just for fun!

Each week there will be a “math puzzler” for the class to think about. Please feel free to submit a “puzzler” you think the class might enjoy. The “puzzlers” don’t have to be difficult, nor related to the material in class — they just have to be fun to think about!

More from the *Island of Dreamers*. (This problem is from Math Horizons Sept 2005 issue. It is a great logic puzzle from a book by Raymond Smullyan.)

In HW 6 we were introduced to the Island of Dreamers. Recall that on this island there are two kinds of people — diurnal and nocturnal. Diurnals are characterized by this property: everything they believe while awake is true while everything they believe while asleep is false. Conversely, everything nocturnals believe while sleeping is true and everything nocturnals believe while awake is false.

I tell a logic expert the following: “An inhabitant on the Island of Dreamers once believed that he was diurnal and awake.” What was he? The expert responds to me (correctly) that there is not enough information to solve the problem. The expert then asks me if I know what type the person is and whether he was awake or asleep at the time. I respond that I do. The expert then asks me the following question: “If you told me whether he was diurnal or nocturnal, would I have enough information to solve the problem?” I answer truthfully and the expert then proceeds to solve the problem.

Question for you: What was the person really?

This is an example of a metapuzzle (puzzles about puzzles). A metapuzzle is a logic puzzle that doesn’t come with enough information to solve the puzzle. But then we are given some extra information (often indirectly) that allows us to solve the puzzle. Another example of this is the puzzler I gave you on HW 2 the gold star problem. No one could work out if they had a gold star on their cap, but the extra information given by the teacher in the first round and then given by each other in subsequent rounds allowed people to solve the puzzle.