

# Math 25b Homework 10

Due Wednesday 19th April 2006.

Half of this problem set will be graded by Alison and half by Ivan. Please turn in problems from Section 1 separately from the problems in Section 2. Remember to staple each bundle of solutions and also to put your name on each!

## 1 Alison's problems

(1) *Volumes and Determinants*

Problem 3–35 on page 62 of Spivak.

(2) *Switching the order of integration*

Compute

$$\int_{x=0}^{x=1} \int_{y=x}^{y=1} e^{y^2} dy dx$$

by writing it as an iterated integral in the other order: integrating  $x$  first and then  $y$ . (Warning: this is *not* equal to  $\int_{y=x}^{y=1} \int_{x=0}^{x=1} e^{y^2} dx dy$ .)

(3) *Switching the order of integration*

(a) The following iterated integral finds the volume of a certain solid in  $\mathbb{R}^3$ .

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} 1 dy dz dx$$

Draw a sketch of the solid in question.

(b) Now rewrite this integral as an iterated integral in the five other orders of integration. (You do not have to evaluate this integral.)

(4) *Is this obvious to you? (Apparently I'm not a mathematician....)*

Parts (d) and (e) of problem 3–41 on pages 73–74 of Spivak.

## 2 Ivan's problems

(1) *Cylindrical and Spherical coordinates*

(a) Find out what the cylindrical and spherical coordinate systems are on  $\mathbb{R}^3$ . (So describe what they are and how you might go between the new coordinates and rectangular coordinates. You might also like to give an example of a function in rectangular coordinates and then give it in the new coordinates. If you pick the right function, you'll see why the coordinates have the names they do.)

(b) Use the change of variable formula to derive analogs of the relation

$$"dx dy = r dr d\theta".$$

(c) Find the volume of the solid that lies inside the sphere  $x^2 + y^2 + z^2 = 5$  and lies above the paraboloid  $x^2 + y^2 = 4z$ .

(d) Find the mass of the solid lying between two concentric half spheres of radii  $a$  and  $b$ , where  $0 < a < b$ , if the density at each point is equal to the square of the distance of that point from the center.

(2) *Clairaut's Theorem*

Problem 3-28 on page 61 of Spivak.

**(Remark:** use Spivak's suggested method, but note that you can prove this theorem directly using a little thought and the mean value theorem. Many multivariable textbooks include this proof either in the text or in an appendix.)

(3) *Change of variable*

(a) Evaluate

$$\int \int_R (x + y)e^{-x^2 - y^2} dx dy$$

where  $R \subset \mathbb{R}^2$  is the rectangle enclosed by the lines  $x - y = 0$ ,  $x - y = 2$ ,  $x + y = 0$  and  $x + y = 3$ .

(b) Find the area bounded by the ellipse  $9x^2 + 4y^2 = 1$ . (Can you generalise this?)

(4) *We need something more than tensor products...*

Let  $V = \mathbb{R}^4$  considered as a vector space over  $\mathbb{R}$ . Suppose  $V$  has basis  $e_1, e_2, e_3, e_4$  and the dual space  $V^*$  has basis  $f^1, f^2, f^3, f^4$ . (Recall  $f^i(v_j) = \delta_j^i$ .)

(a) Let  $w = f^1 \otimes f^2 - f^2 \otimes f^1$ . Show that  $w \in \Omega^2(V)$  (is an alternating tensor).

(b) Show that  $w \otimes w$  is not alternating. (Hint: what happens when you apply  $w \otimes w$  to  $(e_1, e_2, e_1, e_2)$ ?)