Math 25b Homework 7

Due Friday 24th March 2006.

Half of this problem set will be graded by Alison and half by Ivan. Please turn in problems from Section 1 separately from the problems in Section 2. Remember to staple each bundle of solutions and also to put your name on each!

1 Alison's problems

(1) *A lemma used repeatedly in Spivak.*
(a) Problem 1-10 on page 5 of Spivak.
(b) Show that any linear map $T : \mathbb{R}^m \to \mathbb{R}^n$ is continuous.
Note: There are linear maps between infinite-dimensional inner product spaces which are not continuous.

(2) *Differentiability implies continuity.*
Problem 2-1 on page 17 of Spivak.

(3) *Computing derivatives.*
(a) Problem 2-10 parts (b) and (c) on page 23 of Spivak.
(b) Problem 2-11 parts (b) on page 23 of Spivak.

(4) *Differentiating bilinear functions.*
Problem 2-12 on page 23 of Spivak.

(5) *Differentiating inner products.*
Problem 2-13 on page 23 of Spivak.

2 Ivan's problems

(1) *Computing partial derivatives.*
Problem 2-17 parts (b), (g) and (h) on page 28 of Spivak.

(2) Problem 2-21 on page 28 of Spivak.

(3) *Partial derivatives of differentiable functions need not be continuous.*
Problem 2-32 on page 33—34 of Spivak. (Note that we have done part (a) earlier in this semester — see example 5.6 on page 106 of Rudin.)

(4) *The existence of partial derivatives does not imply differentiability.*
Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$:

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$

(a) Show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist everywhere on $\mathbb{R}^2$.
(b) At what points $(x,y) \in \mathbb{R}^2$ is f differentiable?
You should figure out why your answer to (b) does not contradict Theorem 2.8 on page 31 of Spivak.

(5) *Chain rule.*
(a) Problem 2-28 part (b) on page 33 of Spivak.
(b) The temperature at a point (x,y) in the plane is given by $T(x,y) = x^2y + 3xy^4$. An ant crawls on the plan such that its position after t seconds is given by $x = \sin 2t$ and $y = \cos t$. Find the rate of change of temperature along the ant's path when $t = 0$.

3 The problems I didn't assign—good practice!

(1) Take a look at problem 2-26 on page 29 of Spivak. This should look familiar to HW 3 problem 3 (in Ivan'ts problems).
(2) Note that all the starred (*) problems in Spivak are ones which are used later in the text or in other problems. Many of them we have already done (especially in Chapter 1), but take a look at them anyway.