Notation used in Math 25a

(Last modified Friday 30th September 2005.)

1 Common abbreviations

"iff" means if and only if

"Def'n" is my shorthand for Definition. "Def" is also often used.

"Thm" is a common shorthand for Theorem

"Cor" is a common shorthand for Corollary

"nbhd" common shorthand for neighbourhood. Sometimes I write "n'hood" or "nhood".

"cts" is my shorthand for continuous

"T.F.A.E" means the following are equivalent

"bded" is my shorthand for bounded.

2 Sets

$\mathbb{N}$ natural numbers $0, 1, 2, 3, \ldots$

$\mathbb{Z}$ integers

$\mathbb{Q}$ rationals

$\mathbb{R}$ reals

A, B, C, X, Y, $\ldots$ often denote sets

Elements of sets often are not capiltalized, example $a \in A$.

$a \in A$ means a is an element of the set A.

$a \notin A$ means a is not an element of the set A.

$\emptyset$ is the emptyset.

$A \subset B$ or $A \subseteq B$ means A is a subset of B. I'm not sure there is a standard notation for a proper subset (one that is not empty, nor the whole set). So if you are assuming this, it is better to state this in words.

$A \not\subseteq B$ means A is not a subset of B

$A \times B$ means the set of all (a, b) with $a \in A$ and $b \in B$

A^2 means $A \times A$ and A^n means $A \times \ldots \times A$ (n copies of A)

$\{x \in X | x > 1\}$ means the set of all x in X such that $x > 1$

$\{x \in \mathbb{R} : 5x < 11\}$ means the set of all real x such that $5x$ is less than 11.

$:=$ means is defined to be. For example $[a] := \{b \in A : a \sim b$ reads "$[a]$ (an equivalence class) is defined to be the set of all $b \in A$ such that b is equivalent to a". For example $A/\sim := \{[a] | a \in A\}$ reads "$A/\sim$ is defined to be the set of all $[a]$ such that a is in A".

3 Functions

$f : A \to B$ means the function f from set A to set B

$x \mapsto f(x)$ means the element $x \in A$ maps to the element $f(x) \in B$. So we use "$\to$" to indicate the map between the sets and "$\mapsto$" to show where are particular element is going.

For example take a familiar function $y = x^2$. Let's write this "properly". So we'd have $f : \mathbb{R} \to \mathbb{R}$ and $x \mapsto x^2$. We also write the latter as $f(x) = x^2$.

4 Quantifiers and Implications

$\forall$ means for all. For example $\forall x \in \mathbb{R}$ means for all real numbers...

$\exists$ means there exists. For example an integer $k \in \mathbb{Z}$ is divisible by 7 if $\exists p \in \mathbb{Z}$ such that $k = 7p$. So we are just claiming the existence of one (or more) such integers in this case.

! means unique — usually as seen in the next item

$\exists!$ means there exists a unique — for example "$\exists! x \in \mathbb{R}$ such that..." reads there exists a unique real x such that.... The proof of this statement will have two parts. The first proving existence, the second uniqueness. In my experience usually one is easier to prove than the other.

$P \Rightarrow Q$ means if P then Q. Note that this is logically equivalent to the contrapositive statement $not(Q) \Rightarrow not(P)$.

$P \iff Q$ means P if and only if Q. Note that you need to prove two statements here, $P \Rightarrow Q$ and $Q \Rightarrow P$.

5 Others

$m|n$ means m divides n. For example $2|10$, but $3 \not| 16$.

$\sum$ means take the sum of — for example $\sum_{k=1}^{20} k^2 = 1^2 + 2^2 + \ldots + 20^2$.

Π means take the product of — for example $\Pi_{k=1}^{10} k = 1 \cdot 2 \cdot \ldots \cdot 10 = 10!$