Proof writing

(A short meditation on writing good proofs.)

1 Preamble

A large component of the Math 25 course is mathematical proofs. This means both reading them (text and lectures) and writing them for yourselves (HW assignments). In the Fall semester we've learned about making arguments rigorous. (The fact that you do need to justify steps and many things are not as obvious as you might initially think.) As a consequence many of your HW sets have gone from having too little detail to being correct and having too much detail. Our aim this semester is to work on your writing style. Now you can write correct proofs, let's make them elegant as well!

As a reminder, good proofs are:

- Correct — ideally, every statement should follow from axioms or from what has been proved before.
- Concise — a proof should not contain anything that is not necessary.
- Readable — Human beings both write and read proofs. Don't be afraid to explain in words what you are doing. For example, before embarking on a long computation, it is a good idea to explain what you are doing and why you are doing it.

How can you achieve this? It is after all a bit subjective. When you are new to a subject, you will want to write in more details. You also need to get the hang of the style or flavor of the standard arguments. From talking to you, I think many of you find Rudin's proofs on the short side (terse, everything is there, but you really have to think about it and maybe an extra sentence or two wouldn't hurt). On the other hand many of you found Axler's proofs to be wordy (everything is there, but you don't really need all the steps he puts in). Perhaps aim for your proofs to be somewhere between the two?

Here are some things to think about as you solve your problems and write things up. After you solve a problem **rethink the problem again and rethink your solution**. Ask yourself the following quesitions:

1. What are the key steps in the solution?
2. Have I tried to do two different proofs at the same time?

3. What are the steps filling in the details?

4. Have I given too much detail? Is there are shorter way to justify the key steps?

5. Is there a separate result that is buried in the solution that could be written separately?

Now rewrite your solution.

1. You want to draw attention to the key steps so it is easy to follow your arguments. (This may mean writing another result up first, before giving your main argument.)

2. Think about the layout of your proof on the page. Don't squash it up, don't hesitate to start new lines.

3. Make sure you are not sloppy with your notation—write what you mean!

4. Make sure you do have enough justification in your slimmed down proof!!

2 HW sets and solutions

Learning how to write proofs both correctly and elegantly takes time. We want to give you some practical help with this. Each week one problem on the HW will be graded in a special way. You will be given two grades. One for correctness (as we do for the other problems) and one for style (for example, have you given us a wandering essay, or have you written to the point). If your style grade is lower than your correctness grade you'll get comments explaining what can be done to improve it.

We are also going to write up the HW solutions a little differently. If the entire class gets a problem correct, then we will not give a solution for it. If everyone gets a problem correct, but for a silly little detail, then in the solutions we'll make a comment about this detail. We will give complete solutions to the rest of the problems (especially the one graded for style).

I hope these guidelines are helpful. If you are confused about any of these issues, please either email me or ask me in person.