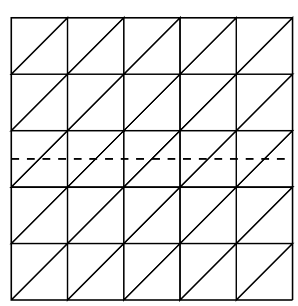
Math 272a, homework 1

September 19, 2003

Exercises from Hatcher's book. Please do questions 1, 5, 8 and 14 from section 2.1. These questions will require that you read and understand Hatcher's definition of a Δ-complex, something more general than a simplicial complex.

Other problems

Problem 1. The following picture is a triangulation of a square (ignore the dotted line). Identifying opposite edges of the square with reversed orientation, we get a triangulation K for the space $\mathbf{RP}^2$, the real projective plane. Order the vertices of the corresponding abstract simplicial complex K as you will, and describe a 1-cycle representing a non-zero element of $H_1(K)$. Use the dotted line to prove that your 1-cycle is not a boundary.



Problem 2. Let K be an abstract simplicial complex with vertices V and simplices S. The *cone* on K is the abstract simplicial complex CK whose vertex set is $V \cup \{*\}$ and set of simplices is the union of:

(a) the simplices S of K;

(b) the simplicies $s \cup \{*\}$ for $s \in S$;

(c) the single-element set containing the 0-simplex $[*]$.

Show that the homology of the abstract simplicial complex CK is zero in all positive dimensions.

Problem 3. Define relative homology for a general pair (K, K') consisting of a complex and a subcomplex. Let K be the unique abstract simplicial complex with exactly one $(n+1)$-simplex and $n+2$ vertices $\{v_0, \dots, v_{n+1}\}$. Let K' be the subcomplex which is its n-skeleton, so $|K'|$ is homeomorphic to an n-dimensional sphere. Use the long exact sequence of a pair to prove that $H_n(K') = \mathbf{Z}$ if $n \geq 1$. (You might want to use the previous problem.)

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