Math 272b, homework 8

April 18, 2004

Problem 1. We determined $KO(\mathbf{R}^n)$ (and thus $KO^{-n}(\text{point})$) for all n. Starting with $n = 1$, it goes: "zee-two zee-two zero zee; zero zero zero zee."

Determine the *ring* structure of $KO^*(\text{point})$.

Problem 2. Show that every complex vector bundle of rank $8k$ on a sphere S^{8k} is of the form $E \otimes_{\mathbf{R}} \mathbf{C}$, for some real vector bundle of real rank $8k$.

Problem 3. Calculate the Pontryagin class p_{2k} for the Bott generator of $KO(S^{8k})$.

Problem 4. Use the 24-term exact sequence of the pair $(\mathbf{RP}^2, \mathbf{RP}^1)$, together with Bott's calculation of $KO(S^n)$, to calculate the group $KO^n(\mathbf{RP}^2)$ for all n. (You might find that the exact sequence leaves some ambiguity concerning the case $n = 0$; resolve the ambiguity using the Stiefel-Whitney classes.)

Problem 5. *Optional.* What is the ring structure on $\oplus_n KO^n(\mathbf{RP}^2)$? What is the map $KO^n(\mathbf{RP}^2) \to KO^n(S^2)$ that one obtains from the double-covering $S^2 \to \mathbf{RP}^2$?

Peter Kronheimer