Math S-302 Theory and Practice of Teaching Geometry

M-F 9:30 am-12 noon, Science Center, Room 216, July 5-July 21

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Text Materials: Excerpts from Benesh, Boester, Weinberg, and Wiesner's Text.

Math S-302 is a geometry class that is a part of the "Master of Liberal Arts in Mathematics for Teaching" degree. While I expect that many of the students in this class will be in this program, it is not necessary for you to be enrolled in this program to take the course. There are four goals for this course:

- *To think and communicate mathematically.* This will include sharpening your problem-solving and reasoning skills os that you can solve hard problems that no one has shown you how to do. This is what makes mathematical thinking so powerful: it allows people to solve brand new problems. You will also practice explain mathematical ideas to others, both verbally and in written form. This, too, is an essential part of mathematics. An idea is not very useful if it is stuck inside one person's head!
- *To build a solid mathematical base.* In particular, our goal is to increase the depth of our understanding of Euclidean geometry - the usual geometry that one learns in high school. Note that the emphasis is on depth - we will be not be concentrating on learning a lot of new facts, but rather we will be learning why many of the facts that are commonly known are true. To do this, we will be studying Taxicab geometry. Taxicab geometry is interesting in its own right, but we will be using it mainly to enhance our understanding of Euclidean geometry. Taxicab geometry will do this by giving us something to which we can compare and contrast Euclidean geometry.
- *To practice and experience new teaching experiences.* The course name is "Theory and Practice of Teaching Geometry," so there should be a teaching component of this course. And...there is. In fact, the teaching component is two-fold. The first fold is that the course will be taught in a manner that is suggested by the creators of the *Core Plus* and *Connected Mathematics Project* curricula. This will be described in greater detail below. The second teaching component is that you will be doing a large share of the teaching. This will be a student-centered classroom, and you will be expected to act as both the teacher and the learner in both small group and large group environments.
- *To have fun.* Believe it or not, people used to do mathematics as a hobby. This is because mathematics, when done correctly, can be fun. Uncovering the problem to a hard math problem can be very rewarding, as can figuring out the exact right way to make someone else understand a mathematical idea. This goal is highly intertwined with and supports the previous three goals.

A note about the structure of this class.

In order to achieve these goals, you must *do* mathematics. This means trying hard problems, struggling with new ideas, and sharing your ideas with others. Because of this, you will spend most class time working in small groups on problems found in the course pack. This begs the question: "If you are going to be solving all of the problems in class, what good am I, the course instructor? What is my role?" I will be there to act as a guide, helping you to focus your learning experience. As an analogy, think of a sports coach or music instructor. In order to learn how to play the piano, you have to actually play the piano. However, a piano teacher helps you learn to play better by watching you play, noticing mistakes, and offering ways you might improve your technique. Just as a piano teacher cannot help you learn to play the piano by playing for you, I cannot help you learn by doing the math for you. This means that instead of telling you how to do a problem or giving you answers, I will guide you toward your own solutions and help you decide for yourself whether or not the answer is valid.

We will also spend time discussing, as a class, our results and difficulties with various problems. This will give you a chance to communicate your ideas to others and to see how others have approached or solved a given problem.

In addition to in-class work, there will be out-of-class work. This will sometimes take the form of a normal problem set, and other times take the form of a paper (sentences, paragraphs, and everything).

Just to be completely clear, you will *not* be expected to play the piano in this class.

Course Expectations and Grades

Below is an outline of what you are expected to do in this class, and how I will determine your grade.

- Attendance and Participation (15 percent)

  Because so much of the learning in this class will occur in the classroom, it is essential that you come to class, actively engage in the work of your group, and participate in class discussions. You will be expected to stay on task, work hard on the assigned problem, engage in class discussions, and *help your fellow group members and classmates*. The last item requires a little explanation: if you already know the solution to the problem, you are still expected to help the group by offering explanations and asking good questions. If you are completely confused by a problem, you are expected to help the group by asking questions to help lessen your confusion.

- Homework (50 percent)

  In addition to working on problems in class, I will ask you to write up your results for certain problems. These reports will include a description of the problem, your solution to the problem, and an explanation of why the solution works. These problem reports will sometimes be done individually, and will sometimes be done as a group. Each time you are assigned a written assignment, you will receive a rubric describing how I will be grading the assignment. Use this to help you write the paper.

This course is *not* graded on a curve. In other words, it is to your benefit to help your classmates out as much as possible - if they get a higher grade, your grade will not be lowered at all.

You will have a couple of problem sets (i.e. "Normal math homework") assigned during this course.

- Exams (35 percent)

  There will be two exams:

  - Midterm: In class, Thursday, July 13.
  - Final Exam: 9 am, Thursday, July 20.

  These exams will be based on the work we do in class. You will need a thorough understanding of the material we have worked on in class, and many of the problems will likely be ones you have not seen before. However, the problems will be such that you can solve them in a given time on your own.

Course Outline

Here is a list of the topics we will cover this semester:

- An introduction to taxicab geometry
- Areas, volumes, and measurement
- Rigid Motions
- Symmetry
- Tessellations
- Constructions