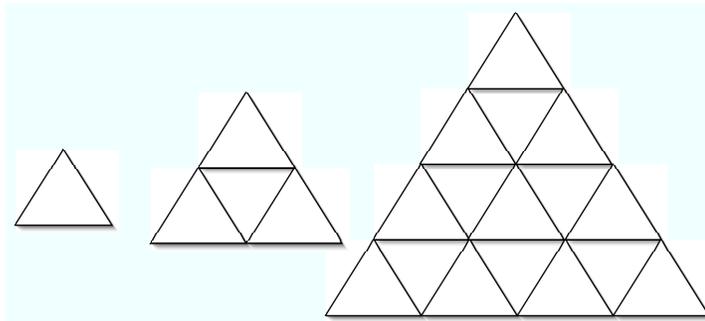


Homework Set #4 (30 points, 54 available)

Read “A Brief History of Algebraic Notation” by Lynn Stallings.

On Monday we will begin talking about the use of variables in algebra. If you have materials or manipulatives which you have used to teach students about variables, bring them to class with you on Monday.

- (6 points) On a string of beads, the largest bead (or beads) is in the center, and the smaller ones are on the two ends. Each of the small beads on the two ends costs \$1; each of the next larger beads costs \$2; the third bead from each end costs \$3; and so on. On the basis of this plan, how much would a string of 9 beads cost? 12? 25? n ?
- (8 points, each part is worth 2 points) The first diagram below is made up of three line segments of equal length. You can think of the second diagram as made up of nine segments of that same length, and so on.
 - Find the number of segments of the given length that would be needed for the tenth diagram in this sequence.
 - Explain how you would find the number of segments of the given length that would be needed for the 100th diagram in this sequence. (You don't have to number, just describe how you would find it).
 - Find a recursive formula that gives the number of segments for the n th diagram in the sequence.
 - Find a closed form formula for the number of segments in the n th diagram in the sequence.



- (6 points) Which conclusions are true, false, or undetermined by the following if-then statement: “If it rains, then it pours“? Explain each of your answers briefly (1-2 sentences).
 - If it does not rain, then it won't pour.
 - If it does not pour, then it won't rain.
 - If it pours, then it rains.
 - It rained, but it didn't pour.
 - It poured, but it didn't rain.
 - It neither rained nor poured.
 - It rained and it poured.
- (8 points) Make addition and multiplication tables for mod 2, mod 3 and mod 4 arithmetic.

5. (8 points) Make a conjecture about when you can do the division problem $a \div b$ in mod m arithmetic (you may find the previous problem helpful in trying to make your conjecture especially if you also look at multiplication mod 5 and 6, or you can just make up your own examples). Provide some supporting examples (and non-examples) for your conjecture. You could also try to prove your conjecture, but it's not necessary.
6. (8 points) Consider the set of ordered pairs $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$. We define a way to add and multiply these ordered pairs:

$$(a, b) + (c, d) = (a + c \pmod{2}, b + d \pmod{2})$$
$$(a, b) \cdot (c, d) = (ac \pmod{2}, bd \pmod{2})$$

Which of the Rules of Arithmetic are satisfied by this addition and multiplication? For each property that is satisfied, you don't have to give a full proof, but give a brief statement or argument about why (one sentence is fine). For each property that is not satisfied, give a counterexample.

7. (5 points) You might wonder if it is ever possible to have a method of addition or multiplication which is not associative. In fact, floating-point arithmetic, which is used by computers, is not associative! Read about floating point arithmetic (Wikipedia's article at http://en.wikipedia.org/wiki/Floating_point is a good choice). Give a couple of examples (other than the one given in the Wikipedia article) of cases where floating point arithmetic proves to be non-associative on a computer (for example, using Excel or another spreadsheet program or a system for mathematics like Mathematica or Maple) or on a handheld calculator (you should actually try out each example and report back what the computer or calculator tells you).
8. (5 points) Using the most powerful telescope ever made, scientists observe a class of young aliens on a planet millions of light years away. On the blackboard, their teacher has written these equations:

$$13 + 14 = 30$$
$$3 \times 3 \times 3 \times 3 = 144$$

How many fingers do the aliens have? (We count in base ten because we have 10 fingers). The idea for this question came from Ask Dr. Math. <http://mathforum.org/library/drmath/view/57148.html> (so if you need a hint about how to solve the problem, you can go there!)