Math 55a: Honors Advanced Calculus and Linear Algebra

Homework Assignment #8 (8 November 2002):
Linear Algebra IV — "Eigenstuff"

HINT, *n.*: The hardest of several possible ways to do a proof.[1]

1.–10. Solve Exercises 4, 7–12, 15, 16, 21 from Chapter 5 of the textbook (pages 94,95). As usual, $\mathbf{F}$ can be any field, and $\mathbf{C}$ can be any algebraically closed field. Do not assume that vector spaces are finite dimensional unless you must. For #4, remember that Axler's "null$\,(T)$" is our "ker$\,(T)$". For #16, how much of #15 remains true over an arbitrary field?

[#15 has the following important consequence: if $P \in F[z]$ and $P(T) = 0$ for some linear operator $T \in \mathcal{L}(V)$, then every eigenvalue of T is a root of P. For instance, the only possible eigenvalues of a linear involution are ± 1, the roots of $z^2 - 1$.]

For the next computational problem, make sure to check your answer against the actual entries of A^t for the first few t.

11. Let A be the 2×2 matrix $\frac{1}{7}\left[\begin{smallmatrix} 6 & 3 \\ 2 & 5 \end{smallmatrix}\right]$. Find a closed form for (the entries of) A^t as functions of $t = 0, 1, 2, \ldots$. [Hint: Begin by finding the eigenvalues and eigenvectors of the linear transformation corresponding to A.] What happens to A^t asymptotically as $t \to \infty$? What happens if A is replaced by the matrix $\left[\begin{smallmatrix} 0.2 & 1.2 \\ -0.6 & 1.4 \end{smallmatrix}\right]$?

This problem set is due Monday [sic], 18 November, at the beginning of class.

[1] *Definitions of Terms Commonly Used in Higher Math*, R. Glover et al.; cf. also Prob. 11.