Math 55a, Fall 2004

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8 th Assignment, due November 16

Because of the midterm and the November 11 holiday, his assignment is "lighter" than most of the previous assignments.

1. Recall the definition of the quotient field of an integral domain D: a field K, containing D as sub-ring-with-unit, such that there does not exist a proper subfield of K which contains D.
a) Prove that every integral domain has a quotient field, and that the quotient field is unique up to isomorphism.
b) Identify the quotient field of $\mathbb{Z}$ with $\mathbb{Q}$.
c) Let K be a field, $K[X]$ the ring of polynomials over K. What is the quotient field of $K[X]$? A short answer suffices.

2. In this problem R denotes a commutative ring, $\{M_\alpha \mid \alpha \in A\}$ a collection of R-modules, and N another R-module. Show that $\mathrm{Hom}(\oplus_{\alpha\in A} M_\alpha, N)$ is canonically isomorphic (i.e., independently of arbitrary choices) to the direct product of R-modules $\Pi_{\alpha\in A}\,\mathrm{Hom}(M_\alpha, N)$.