Math 55a, Fall 2004

First Assignment, due September 28

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1. In lecture, we gave an informal description of a particular bijective map $F : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$. Write out an explicit formula for a map of this type, or for the inverse of such a map. Show that your map has the required properties.

2. Let X be a non-empty countable set. Prove the following two assertions:
a) X has the same cardinality as $\mathbb{N}$ if and only if there exists a non-empty, proper subset $Y \subset X$ such that $\operatorname{card}(Y) = \operatorname{card}(X)$.
b) If X does not contain a subset $Y \subset X$ of this type, there exists a unique integer $n \geq 1$ such that $\operatorname{card}(X) = \operatorname{card}\{1, 2, \ldots n\}$.

3. Let n be an integer strictly greater than one. Construct a surjective map from $\operatorname{Mor}(\mathbb{N}, \{1, 2, \ldots, n\})$ to the set of non-negative real numbers. Either show that your map is also injective, or analyze the degree to which it fails to be injective. Deduce that $\mathbb{R}$, the set of real numbers, has the same cardinality as the power set of $\mathbb{N}$, i.e., the same cardinality as $\operatorname{Mor}(\mathbb{N}, \{0, 1\})$.