ARCHIMEDES

BY

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With a new bibliographic essay by Wilbur R. Knorr

Summis ingeniis dux et magister fuit

(Heiberg, *Archimedis opera omnia* III,
Prolegomena XCV)

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PREFACE

This book is an attempt to bring the work of Archimedes, which is one of the high-water marks of the mathematical culture of Greek Antiquity, nearer to the understanding and the appreciation of the modern reader. Such an attempt has been made twice before, in a way so excellent that I can scarcely hope to equal it: by T. L. Heath in *The Works of Archimedes* and by P. Ver Eecke in *Les Oeuvres Complètes d'Archimède*. My belief that I might be excused for adding to these two excellent editions a new adaptation of the writings of the great Greek mathematician finds its justification in the consideration that the method of treatment here chosen differs fundamentally from the one followed by Heath as well as from that applied by Ver Eecke. As a matter of fact, Heath represents Archimedes' argument in modern notation, Ver Eecke gives a literal translation of his writings. Both methods have their disadvantages: in a representation of Greek proofs in the symbolism of modern algebra it is often precisely the most characteristic qualities of the classical argument which are lost, so that the reader is not sufficiently obliged to enter into the train of thought of the original; the literal translation, on the other hand, which like the Greek text says in words everything that we, spoiled as we have been by the development of mathematical symbolism, can grasp and understand so much more easily in symbols, perhaps helps the present-day reader too little to overcome the peculiar difficulties which are inevitably involved in the reading of the Greek mathematical authors and which certainly are not due exclusively, nay, not even primarily, to the fact that they wrote in Greek.

The method applied in the present book attempts to combine the advantages and avoid the disadvantages of the two methods just outlined. The exposition follows the Greek text closely, but only the propositions are given in a literal translation; after that the proofs are set forth in a symbolical notation specially devised for the purpose, which makes it possible to follow the line of reasoning step by step. This system of notation, which was also used in my work *De Elementen van Euclides* (The Elements of Euclid) (Groningen, 1929, 1931), in long practice has been found a useful aid in the explanation of Greek mathematical arguments.

Apart from the introduction of this aid, I have also tried to meet in another way the difficulties which I know from experience are encountered by present-day mathematicians reading Greek authors. In fact, the Greek mathematicians in their works are wont to give, without a single word of elucidation about the object in view, a dry-as-dust string of propositions and proofs, in which not the slightest distinction is made between lemmas and fundamental theorems, while the general trend of the argument is often very difficult to discover. In order to bring this trend out more clearly I have collected in a separate chapter (Chapter III) all those theorems which in relation to the nucleus of a treatise have the function of elements (*στοιχεῖα*); with each individual work the argument could then be summarized much more briefly, because all the lemmas had already been discussed previously. A decimal classification of Chapter III makes it possible to trace these lemmas quickly, if desired, and to find out how they can be proved. Through this arrangement the additional advantage has been gained that each of Archimedes' treatises can be studied separately.

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0.1 *Notations*:

A rectangle with sides a and b	$\mathbf{O}\,(a, b)$	**O** of Ὀρθογώνιον
A square with side a	$\mathbf{T}\,(a)$	**T** of *Τετράγωνον*
A circle with diameter d	$\mathbf{K}\,(d)$	**K** of *Κύκλος*

The ratio of two homogeneous[1]) magnitudes A and B

$$(A, B)$$

The ratio of two squares with sides a, b, for example, is written:

$$[\mathbf{T}(a), \mathbf{T}(b)]$$

The above symbols are used in particular in the practice of the so-called Application of Areas or Geometrical Algebra, which is the instrument used in Greek analytical geometry, and the elements of which are dealt with in Euclid[2]). If desired, any line of reasoning expressed by these symbols may at once be translated into the non prevalent algebraic notation by the following substitutions:

$$\mathbf{O}(a, b) = ab \qquad \mathbf{T}(a) = a^2 \qquad (A, B) = A : B\,.$$

0.2. *Fundamental concepts of the application of areas*[2]).

0.21. A plane figure $\mathbf{X}$ is said to be applied *parabolically* to a line segment A when a line segment B is constructed such that

$$\mathbf{O}(A, B) = \mathbf{X}\,.$$

0.22. A plane figure $\mathbf{X}$ is said to be applied *elliptically* to a line segment A with defect of the assigned form (Δ, E) when $\mathbf{X}$ is applied parabolically to a line segment $B < A$, so that

$$\mathbf{X} = \mathbf{O}(B, \Gamma) \qquad \text{and also} \qquad (\Gamma, A - B) = (\Delta, E)\,.$$

0.23. A plane figure $\mathbf{X}$ is said to be applied *hyperbolically* to a line segment A with excess of the assigned form (Δ, E) when $\mathbf{X}$ is applied parabolically to a line segment $B > A$, so that

$$\mathbf{X} = \mathbf{O}(B, \Gamma) \qquad \text{and also} \qquad (\Gamma, B - A) = (\Delta, E)\,.$$

0.24. The Greek names for the three above-mentioned operations

[1]) Two magnitudes A and B are called homogeneous when they satisfy the axiom of Eudoxus, *i.e.* when there exist natural numbers m and n such that $m.A > B$ and $n.B > A$.

[2]) *Elements of Euclid* II, 12, 103.

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are successively: παραβολή (parabola), ἔλλειψις (ellipse), and ὑπερβολή (hyperbola).

0.3. *Fundamental concepts of the theory of proportions*[1]).

0.31. When $(A, B) = (B, C)$

we call (A, C) the *duplicate ratio* (διπλασίων λόγος) of (A, B).

Symbol: $(A, C) = \mathbf{\Delta\Lambda}(A, B)$.

In this case $(A, C) = [\mathbf{T}(A), \mathbf{T}(B)]$.

The algebraic equivalent of a duplicate ratio is the square of a ratio. In fact, from $a:b = b:c$ it follows that

$$a:c = a^2:b^2 .$$

0.32. When $(A, B) = (B, C) = (C, D)$

we call (A, D) the *triplicate ratio* (τριπλασίων λόγος) of (A, B).

Symbol: $(A, D) = \mathbf{T\Lambda}(A, B)$.

The algebraic equivalent of a triplicate ratio is the cube of a ratio. In fact, from $a:b = b:c = c:d$ it follows that

$$a:d = a^3:b^3 .$$

0.33. When $(A, B) = (M, N)$ and $(B, C) = (P, Q)$

we call (A, C) the *compound ratio* (συγκείμενος λόγος) of the ratios (M, N) and (P, Q).

The algebraic equivalent of a compound ratio is the product of two ratios. In fact, from $a:b = m:n$ and $b:c = p:q$ it follows that

$$a:c = mp:nq .$$

0.4. *Main operations of the theory of proportions*[2]).

0.41. From a proportion $(A, B) = (C, D)$ there result, through the operations to be mentioned below, the proportions given behind them:

[1]) *Elements of Euclid* II, 82–83.
[2]) *Elements of Euclid* II, 71–76.