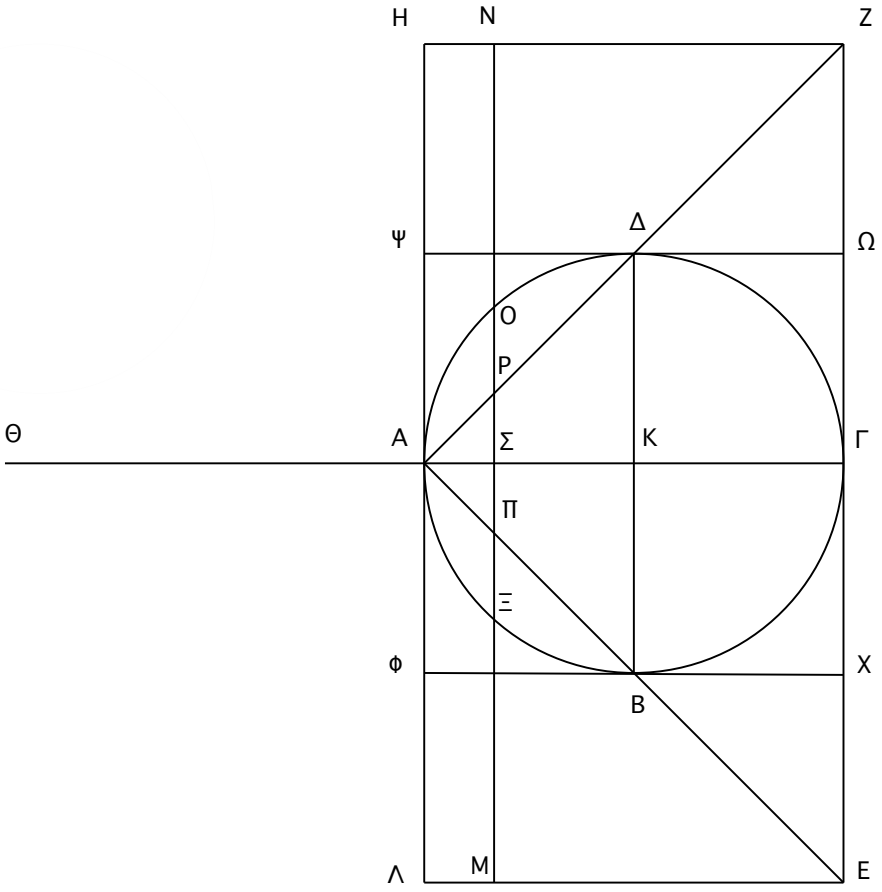
II.

That any sphere is four times the cone whose base is equal to the largest circle of those in the sphere and whose altitude is equal to the radius of the sphere, and that the cylinder whose base is equal to the largest circle of those in the sphere and whose altitude is equal to the diameter of the sphere is one and a half times as large as the sphere, is seen by this method in the following way.

For let there be a sphere in which the largest circle is ΑΒΓΔ and diameters ΑΓ, ΒΔ at right angles to one another; and let there be a circle in the sphere around the diameter ΒΔ perpendicular to the circle ΑΒΓΔ, and from this perpendicular circle let there be a cone drawn having point Α as its vertex; with its surface produced let the cone be cut by a plane through Γ parallel to its base: <this will make a circle perpendicular to> ΑΓ, and its diameter [will be] ΕΖ. From this circle let a cylinder be drawn having its axis equal to ΑΓ; let the sides of the cylinder be ΕΛ, ΖΗ. And let ΓΑ be produced, and let ΑΘ be made equal to it, and let ΓΘ be conceived of as a balance and Α as its midpoint, and let a line ΜΝ be drawn parallel to ΒΔ; let this line cut the circle ΑΒΓΔ at Ξ, Ο, the diameter ΑΓ at Σ, the straight line ΑΕ at Π, the line ΑΖ at Ρ; and from the straight line ΜΝ let a plane be erected perpendicular to ΑΓ: this will make a section in the cylinder <that is a circle whose diameter is ΜΝ, in the sphere ΑΒΓΔ> a circle whose diameter is ΞΟ, in the cone ΑΕΖ a circle whose diameter is ΠΡ.



And since the product of ΓA, AΣ is equal to the product of MΣ, ΣΠ (for AΓ is equal to ΣM, and AΣ to ΠΣ), and the square on AΞ is equal to the product of ΓA, AΣ, i.e. the squares on ΞΣ, ΣΠ, therefore the product of MΣ, ΣΠ is equal to the squares on ΞΣ, ΣΠ. And since, as ΓA is to AΣ, so MΣ is to ΣΠ, and ΓA is equal to AΘ, therefore as ΘA is to AΣ, MΣ is to ΣΠ, i.e. the square on MΣ to the product of MΣ, ΣΠ. But the squares on ΞΣ, ΣΠ were shown to be equal to the product of MΣ, ΣΠ; therefore as AΘ is to AΣ, so the square on MΣ is to the squares on ΞΣ, ΣΠ.

But as the square on MΣ is to the squares on ΞΣ, ΣΠ, so the square on MN is to the squares on ΞO, ΠP, and as the square on MN is to the squares on ΞO, ΠP, so the circle in the cylinder, whose diameter is MN, is to <both the circles, the one in the cone whose diameter is ΠP> and the one in the sphere, whose diameter is ΞO. Therefore as ΘA is to AΣ, so the circle in the cylinder is to the circles in the sphere and in the cone. Now since, as ΘA is to AΣ, so the very circle in the cylinder[25] ... remaining where it is, will be in equilibrium[26] at the point A with both the circles whose diameters are ΞO and ΠP, if they are transferred and placed at Θ in such a way that Θ is the center of gravity of each of them. Similarly it will be shown that also if another line is drawn in the parallelogram ΛZ parallel to EZ, and from the line drawn a plane is erected perpendicular to AΓ, the circle that is created in the cylinder will be in equilibrium around the point A, remaining where it is, with both the circles, the one created in the sphere and the one in the cone, if these are transferred and placed on the balance at Θ in such a way that the center of gravity of each of them is Θ.

And so, if the cylinder is filled up by the circles so taken, as well as the sphere and the cone, the cylinder will be in equilibrium around the point A, remaining where it is, with both the sphere and the cone, transferred and placed on the balance at Θ in such a way that the center of gravity of each of them is Θ. Thus, since the aforesaid solids are in equilibrium at the point A if the cylinder remains around the center of gravity K, while the sphere and cone have been transferred, as has been said, around the center of gravity Θ, as ΘA is to AK, so the cylinder will be to the sphere and the cone. But ΘA is double AK; therefore the cylinder too is double both the sphere and the cone together. And it is three times the cone itself; therefore three cones are equal to two of the same cones and two spheres. Let the two common cones be taken away; therefore one cone, the one having AEZ as the triangle through its axis, is equal to the aforesaid two spheres. But the cone that has AEZ as the triangle through its axis is equal to eight cones which have ABΔ as the triangle through their axes, because EZ is double BΔ. Therefore the eight aforesaid cones are equal to two spheres. Therefore the sphere whose largest circle is ABΓΔ is four times the cone whose vertex is the point A, and whose base is the circle around diameter BΔ perpendicular to AΓ.[27]

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[25] Something has dropped out here (cf. Heiberg 443 n.1). Presumably the original text continued by repeating the rest of the proportion from the previous clause, then going on with the statement about the circle in the cylinder being in equilibrium with the two other circles if they are transferred to Θ.

[26] The verb (*isorropêsousin*) is plural, which does not agree with the singular subject in the text as transmitted.

[27] *On the sphere and cylinder* I, 34.

Now let the lines ΦBX, ΨΔΩ be drawn through the points B, Δ in the parallelogram ΛZ parallel to AΓ, and let a cylinder be conceived of, whose bases are the circles around the diameters ΦΨ, XΩ, and whose axis is AΓ. Since, then, the cylinder with ΦΩ as the parallelogram through its axis is double the cylinder <with ΦΔ as the paral>lelogram <through its axis>[28], and this is itself three times the cone with ABΔ as the triangle through its axis, as in the *Elements*[29], therefore the cylinder with ΦΩ as the parallelogram through its axis is six times the cone with ABΔ as the triangle through its axis. But it was shown that the sphere whose largest circle is ABΓΔ is four times the same cone. Therefore the cylinder is one and a half times the sphere.[30] Which was necessary to be shown.

Once this had been seen – that any sphere is four times the cone having as its base the largest circle and its altitude equal to the radius of the sphere – the idea arose that the surface of any sphere is four times the largest circle of those in the sphere[31]: for the supposition was that just as any circle is equal to a triangle having as its base the circumference of the circle, and its altitude equal to the radius of the circle[32], so also any sphere is equal to a cone having as its base the surface of the sphere, and its altitude equal to the radius of the sphere.

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[28] The bracked words in this sentence are contiguous in the Greek.

[29] Heiberg suspects these words to be an interpolation; Archimedes rarely if ever refers to Euclid's *Elements*.

[30] *On the sphere and cylinder* I, 34 coroll.

[31] *On the sphere and cylinder* I, 33.

[32] *On the measurement of the circle* 1.