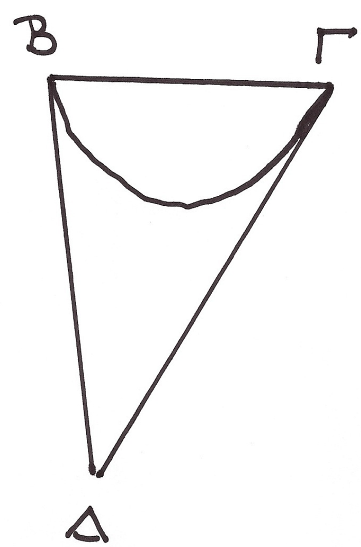
1. About Propositions 14-16 of **Quadrature of the Parabola**

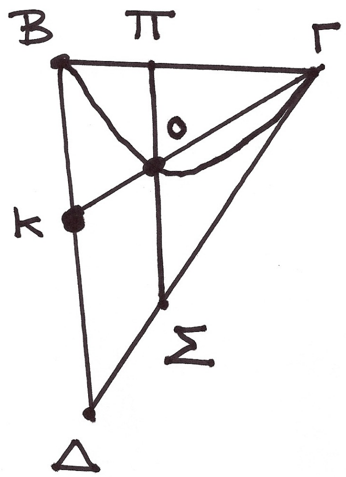
Question 1.1. What is the basic diagram set up in the first paragraph of each of these propositions?

Answer: This diagram:



I'll use the terminology $BO\Gamma$ to denote the pictured segment of a parabola bounded by the straight line $B\Gamma$. As in Archimedes, the straight line $B\Delta$ is drawn parallel to a diameter, and $\Gamma\Delta$ is tangent to the parabola at Γ.

Question 1.2. Return to the basic diagram above. Bisect $B\Delta$ at K. Draw the straight line ΓK.



(1) Why does ΓK intersect the parabolic arc?
(2) Why does it intersect the parabolic arc at exactly one point?
(3) Given that ΓK intersects the parabolic arc at one point, call that point O and draw the straight line through O parallel to a diameter, labeling Π and Σ its intersections with $B\Gamma$ and $B\Delta$, respectively. Is this a special case of the diagrams you are asked to draw in Propositions 14,15, and 16?

Question 1.3. Can you go through the proof of Propositions 14, and 15 using the above diagram as a special case? To begin to do this concentrate on the following figures

- the rectangle $K\Pi$
- the triangles $\Sigma\Pi\Gamma$, $O\Pi\Gamma$, and $B\Gamma\Delta$
- the segment $BO\Gamma$

(1) Show (or rather, note) that

$$O\Pi\Gamma \quad < \quad BO\Gamma \quad < \quad K\Pi \quad + \quad \Sigma\Pi\Gamma.$$

Where, in Archimedes' text of Props 14, 15 are these inequalities acknowledged?
(2) Where and how, in Archimedes' Props 14, 15, do we see the inequalities

$$O\Pi\Gamma \quad < \quad \frac{1}{3}B\Delta\Gamma \quad < \quad K\Pi \quad + \quad \Sigma\Pi\Gamma$$

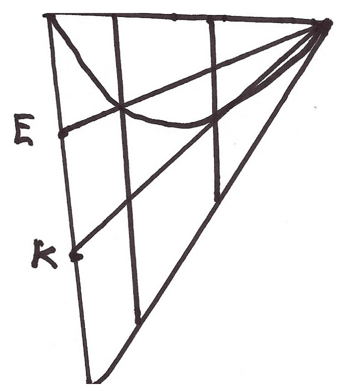
proved?[1]

(3) Note that the difference between $K\Pi + \Sigma\Pi\Gamma$ and $O\Pi\Gamma$ is

$$K\Pi + \Sigma O\Gamma.$$

Why is this difference[2] equal to the triangle $B\Gamma K$?

Question 1.4. Return to the basic diagram above[3] and now *trisect* $B\Delta$ at E and K, so that $BE, EK, K\Delta$ all have the same length. Draw the straight lines $E\Gamma$, $K\Gamma$. Draw the two indicated lines parallel to the diameter of the parabola[4].



(1) Which quadrilaterals in this figure can you show have the same area?
(2) How can you show this?
(3) Which triangles in this figure can you show have the same area?
(4) Where in Archimedes' text is it demonstrated that they do have the same area? How is it demonstrated?
(5) What role does it play in the proofs of these propositions?
(6) Proposition 14, 15 proves inequalities of areas of figures. In the special case we are discussing, state explicitly the inequalities that are being proved.
(7) Run through the proof—in the text of Archimedes' Propositions 14,15—for this case.

[1]In this very particular instance, of course, you can say more precisely what $O\Pi\Gamma$ and $K\Pi + \Sigma\Pi\Gamma$ are, as fractions of $B\Delta\Gamma$.

[2]i.e., the difference between the *upper bound* and the *lower bound* in the chain of inequalities in (1) and (2) above; i.e., the inequalities that sandwich both $BO\Gamma$ and $\frac{1}{3}B\Delta\Gamma$

[3]$BO\Gamma$ is the segment of a parabola bounded by the straight line $B\Gamma$, $B\Delta$ is parallel to a diameter, and $\Gamma\Delta$ is tangent to the parabola at Γ.

[4]These two lines should intersect the parabola at the points at which $E\Gamma$ and $K\Gamma$ (respectively) intersect the parabola.

Question 1.5. Proposition 16 is proved by an indirect argument. Looking at the second paragraph, Archimedes supposes that the equality of areas (asserted by the proposition) is *not true* and he wishes to (eventually) draw a contradiction. He supposes—first— that the area of the parabolic segment is bigger than one third the triangle $B\Delta\Gamma$. How does this lead him to construct the diagram for Proposition 16?

Question 1.6. Make sure that you know exactly the place(s) where—in Props 14, 15, 16—the hypothesis that the parabolic segment is indeed a *parabolic* segment. Where—and exactly how—is it used?

Query 1.7. (For people who know Calculus)

(1) Compare (and contrast) Archimedes' work on this problem with the standard Calculus approach?
(2) For an example of some of the many things to note: Archimedes comes at it with an explicit "indirect argument" formulation (discussed in Question 1.5 above). Do you see clearly what this corresponds to in the standard Calculus approach?

Query 1.8. As has been already noted in class discussions, Archimedes working with his levers, often replaces a figure (or a part of figures such as $\frac{1}{3}B\Delta\Gamma$)with a "space" (*chorion*) whose shape is completely irrelevant but whose area[5]—abstracted from its shape—is considered. Did the word *chorion* play—at least a bit— this role (i.e., meaning a space, but where *area* is of concern and *particular shape* not) earlier?

[5]in other contexts he does the same thing with *volume*, or *length*