

Archimedes

Method concerning mechanical theorems. To Eratosthenes.¹

Archimedes to Eratosthenes: Greetings. Some time ago I sent you some theorems I had discovered, writing down only the propositions because I wished you to find their demonstrations, which I did not give at the time. The propositions of the theorems which I sent you were the following:

1. If in a right prism having a parallelogram² as its base a cylinder is inscribed which has its bases in the opposite parallelograms and its sides on the other planes of the prism, and if a plane is drawn through <the center of the circle> that is the base of the cylinder and one side of the square in the opposite plane, the plane that is drawn will cut off from the cylinder a segment which is bounded by two planes and the surface of the cylinder – the one plane being that which was drawn, the other that in which is the base of the cylinder, and the surface that which is between the said planes. And the segment cut off from the cylinder is one sixth of the whole prism.

The proposition of the other theorem is this:

2. If in a cube a cylinder is inscribed whose bases are against opposite parallelograms and whose surface touches the other four planes, and if in the same cube another cylinder is inscribed whose bases are in other parallelograms and whose surface touches the other four planes, then the figure enclosed by the surfaces of the cylinders and comprehended within both cylinders is two thirds of the whole cube.

These theorems differ from those formerly discovered; for then we compared those figures (conoids, spheroids and <their> segments) with the <figures> of cones and cylinders <in magnitude>³; but none of them was found to be equal to a solid figure enclosed by planes. Each of these figures, on the other hand, [which are enclosed] by two planes and surfaces of cylinders, is found to be equal to a solid figure⁴ enclosed by planes.

Accordingly, I have written down the demonstrations of these theorems in this book and am sending them to you.

¹ Note: this translation was made directly from Heiberg's 1912 text. I have used the following conventions:

[] words added to the translation that are implicit in the Greek

[[]] words deleted by Heiberg

< > words restored by Heiberg

² Here and in the rest of this introduction the 'parallelograms' in question are evidently squares. Cf. Heiberg 428 n.1.

³ In the Greek, the words corresponding to the three bracketed phrases are contiguous.

⁴ Lit. 'to one of the solid figures'

Since I see, however, as I have said, that you are diligent and a prominent teacher of philosophy, and that you value the study of mathematics as the occasion arises, I have thought it well to write down and set out for you in this same book the specific character of a certain method, by means of which it will be possible for you to acquire the ability to study certain mathematical questions by means of mechanics. And I am convinced that this is also no less useful for the demonstration of the theorems themselves. For some things that first became clear to me by means of mechanics were afterwards demonstrated by means of geometry, because the study by this method does not furnish demonstration. For it is easier to supply the demonstration if one has previously obtained some knowledge of what to look for, than to look for it with no prior knowledge.

<..... Hence also with regard to those theorems the demonstrations of which Eudoxos was the first to discover, concerning the cone and the cylinder, that the cone is one-third of the cylinder and the pyramid one-third of the prism that has the same base and altitude, one should give no small share [of the credit] to Democritus, who was the first to make that statement about the said figure⁵ without demonstrating it. In my case also the discovery of the theorem now being published came about in the same way as the earlier [discoveries]; and I have decided to write down and make known the method partly because I have already spoken of it, so that no one would think that I was uttering idle talk, and partly in the conviction that it would be of no small use for mathematics; for I suppose that there will be some among present or future individuals who will discover by the method here set forth still other theorems which have not yet occurred to us.

Accordingly I am writing down first that which also first became clear [to me] by means of mechanics, namely that any segment of a section of a right-angled cone is four thirds of the triangle possessing the same base and equal altitude, and thereafter each of the things that have been seen using the same method. At the end of the book I will write down the geometrical demonstrations of those theorems, the propositions of which I sent <you on an earlier occasion>.

Lemmata

1. If from a magnitude a magnitude is taken away and <the same point is the center of gravity <both of the whole> and of the part removed, then the same point is the <center> of gravity of <the> remaining portion.
2. If <from a magnitude a magnitude <is taken away and> the center of gravity of the whole magnitude and the magnitude removed is not the same point, the center of gravity of the remaining magnitude is on the <straight line> joining the centers of gravity of the whole [magnitude] <and the [magnitude] removed>, if this line is produced and there is taken away from it [a line] which bears the same ratio to the straight line between the aforesaid centers of gravity, as the weight of the magnitude which has been taken away bears to the weight of the remaining magnitude.⁶

⁵ The singular is peculiar, but it seems not to be of particular significance (Dijksterhuis 314 n. 3).

⁶ Demonstrated in *De plan. aequil.* I, 8, where the wording is slightly different (Dijksterhuis 306-7).

3. If the center of gravity of any number of magnitudes is on the same straight line, the center of the magnitude combined from all of them will also be on the same straight line.⁷

4. The center of gravity of any straight line is the midpoint of that line.⁸

5. The center of gravity of any triangle is the point at which the straight lines drawn from the angles of the triangle to the centers of the sides intersect.⁹

6. The center <of gravity> of any parallelogram is <the point at which the diagonals meet.>¹⁰

7. The center of gravity <of a circle> is the point which is also the center of <the circle>.

8. The center of gravity of any cylinder is the midpoint of the axis.

9. The center of gravity of any prism is the midpoint of the axis.¹¹

10. The center of gravity of any cone is on the axis divided in such a way that the segment towards the vertex is three times the remainder.

11. I will also make use of the this theorem [[in the previously written *Conoids*]]¹²:

If any number of magnitudes have the same ratio to the same number of other magnitudes which correspond pair by pair, and if either all or some of the first magnitudes have any ratios whatever to other magnitudes, and the second¹³ magnitudes, the corresponding ones¹⁴, have the same ratios to other magnitudes, then all the first magnitudes have the same ratio to all the said magnitudes¹⁵, as all the second magnitudes bear to all the said magnitudes.¹⁶

⁷ Cf. De plan. aequil. I, 5 (Heiberg 433 n.2).

⁸ Cf. De plan. aequil. I, 4

⁹ De plan. aequil. I, 14

¹⁰ De plan. aequil. I, 10

¹¹ Dijksterhuis 316 n. 1: “As appears from the application in Prop. 13, *axis* here refers to the line segment joining the centers of gravity of the two bases. Archimedes here deviates from his usual terminology, in which ἄξων invariably means the axis of revolution.”

¹² Heiberg excises these words as a marginal gloss that has crept into the text.

¹³ Lit. ‘latter’ (*husteron*).

¹⁴ Taking τὰ ὁμόλογα (434.9 H) with τὰ ὑστερον μεγέθη (434.8 H).

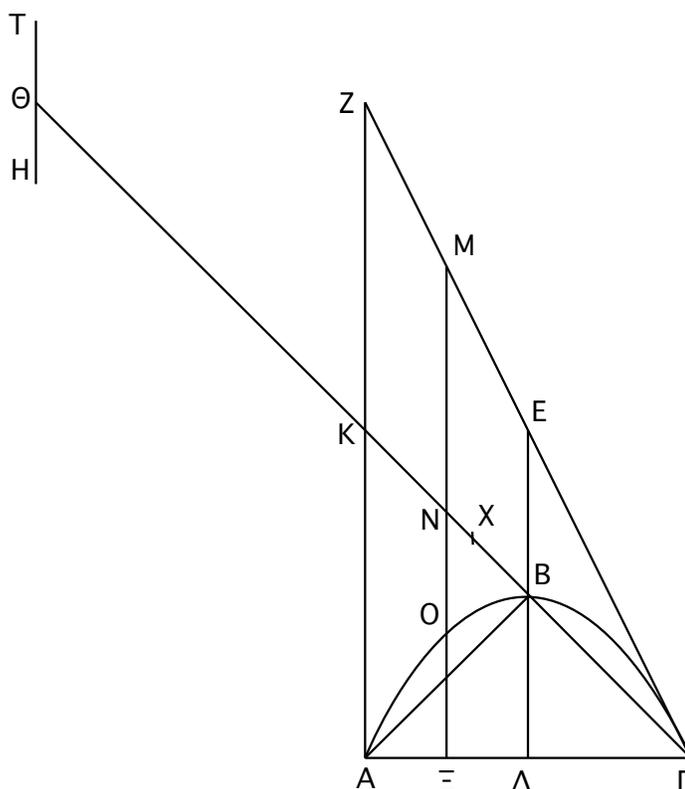
¹⁵ I.e. the magnitudes of the third series, ‘said’ to be in proportion with them.

¹⁶ I.e. the sum of the first series is to the sum of the third, as the sum of the second is to the sum of the fourth. This is demonstrated in *On conoids and spheroids* 1; see Dijksterhuis, 119-122.

I.

Let there be a segment $AB\Gamma$ enclosed by the straight line $A\Gamma$ and the orthotome¹⁷ $AB\Gamma$, and let $A\Gamma$ be cut in two equal parts at Δ , and parallel to the diameter let ΔBE be drawn, and let AB , $B\Gamma$ be joined. I say that segment $AB\Gamma$ is four-thirds the triangle $AB\Gamma$.

From the points A, Γ let there be drawn AZ parallel to ΔBE and ΓZ tangent to the segment, <let ΓB be produced towards K , and let $K\Theta$ be made equal to ΓK >. Let $\Gamma\Theta$ be conceived of as a balance and K as its midpoint, and $M\Xi$ an arbitrary line parallel to $E\Delta$.



¹⁷ This term, borrowed from Dijksterhuis, is shorthand for the phrase ‘section of a right-angled cone’ (*orthogōniou kōnou tomê*): in other words, a parabola (see next n.).

Now since ΓBA is a parabola¹⁸, ΓZ is tangent to it, and $\Gamma \Delta$ is ordinate, EB is equal to $B\Delta$; for this is shown in the *Elements*.¹⁹ For this reason, and because ZA and $M\Xi$ are parallel to $E\Delta$, MN is equal to $N\Xi$ and ZK to KA .²⁰ And since, as ΓA is to $A\Xi$, so $M\Xi$ is to ΞO [[for this is shown in a corollary]]²¹, and as ΓA is to $A\Xi$, so is ΓK to KN ²², and ΓK is equal to $K\Theta$, therefore as ΘK is to KN , so $M\Xi$ is to ΞO .

And since the point N is the center of gravity of the straight line $M\Xi$, because MN is equal to $N\Xi$, if we make TH equal to ΞO and Θ its center of gravity, so that $T\Theta$ is equal to ΘH , $T\Theta H$ will be in equilibrium with $M\Xi$, remaining in its place, because ΘN has been divided in inverse proportion to the weights TH , $M\Xi$, and as ΘK is to KN , so $M\Xi$ is to HT . Hence the center of gravity of the weight combined from each of them is K .

Similarly, whatever lines are drawn in triangle $ZA\Gamma$ parallel to $E\Delta$, they will be in equilibrium, remaining where they are, with the lines cut off from them by the section when these are transferred to Θ , so that the center of gravity of the [weight] combined from them is K .

And because the triangle ΓZA is made up of the lines in the triangle ΓZA , and the segment $AB\Gamma$ is made up of the the lines in the section taken similarly to ΞO , therefore the triangle $ZA\Gamma$ in its present position will be in equilibrium with the segment of the section when this is placed around Θ as its center of gravity, so that K is the center of gravity of the weight combined from each of them.

Now let ΓK be divided at X so that ΓK is three times KX . Then the point X will be the center of gravity of the triangle $AZ\Gamma$: for this has been shown in the *Equilibria*.²³ Now since the triangle $ZA\Gamma$ in its present position is in equilibrium with the segment $BA\Gamma$ at K when it is placed around Θ as its center of gravity, and the center of gravity of the triangle $ZA\Gamma$ is X , therefore as the triangle $AZ\Gamma$ is to the segment $AB\Gamma$ placed around Θ as center, so ΘK is to XK . And ΘK is three times KX : therefore also triangle $AZ\Gamma$ is three times segment $AB\Gamma$. And triangle $ZA\Gamma$ is four times triangle $AB\Gamma$, since ZK is equal to KA , and $A\Delta$ to $\Delta\Gamma$. Therefore segment $AB\Gamma$ is four-thirds triangle $AB\Gamma$. [[Thus this is clear.]]²⁴

Now this has not been demonstrated by what has now been said, but it has given a certain impression that the conclusion is true. And so, since we see that the conclusion has not been dem-

¹⁸ Heiberg (437 n.1) comments that the terminology betrays the hand of an interpolator; the term ‘parabola’ is introduced by Apollonius and not found elsewhere in Archimedes.

¹⁹ I.e. the *Elements of conics* of Euclid and Aristaeus. Cf. *Quadrature of the parabola* 2.

²⁰ Eucl. VI, 4; V, 9.

²¹ Heiberg deems this an interpolation. Cf. QP 5, cor.

²² Eucl. VI, 2; V, 18.

²³ Cf. De plan. aequil. I, 15, 186.3 H with Eutocius, 320.5ff.

²⁴ Heiberg excises this phrase and the entire following paragraph, presumably because it is so repetitious.

onstrated, but suspect that it is true, we shall mention in its proper place the geometrical demonstration which we ourselves found, and which has previously been published.