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The intelligibility of motion and construction: Descartes' early mathematics and metaphysics, 1619–1637

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ABSTRACT

I argue for an interpretation of the connection between Descartes' early mathematics and metaphysics that centers on the standard of geometrical intelligibility that characterizes Descartes' mathematical work during the period 1619 to 1637. This approach remains sensitive to the innovations of Descartes' system of geometry and, I claim, sheds important light on the relationship between his landmark *Geometry* (1637) and his first metaphysics of nature, which is presented in *Le monde* (1633). In particular, I argue that the same standard of clear and distinct motions for construction that allows Descartes to distinguish 'geometric' from 'imaginary' curves in the domain of mathematics is adopted in *Le monde* as Descartes details God's construction of nature. I also show how, on this interpretation, the metaphysics of *Le monde* can fruitfully be brought to bear on Descartes' attempted solution to the Pappus problem, which he presents in Book I of the *Geometry*. My general goal is to show that attention to the standard of intelligibility Descartes invokes in these different areas of inquiry grants us a richer view of the connection between his early mathematics and philosophy than an approach that assumes a common method is what binds his work in these domains together.

1. Introduction

That René Descartes holds an important place in both the history of mathematics and the history of philosophy is hardly a matter of dispute. In the domain of mathematics, he proposed a novel algebraic approach to the study of geometrical curves in his landmark *Geometry* (1637),[1] which helped motivate the development of modern-day analytic geometry. In the domain of philosophy, he promoted a 'rationalist' program of metaphysics and epistemology that shed new light on problems surrounding the existence of God and the human soul, and which directed the course of philosophical investigations in the decades (arguably even the centuries) to follow. Though the historical importance of these Cartesian innovations is uncontroversial, questions remain regarding whether and to what extent Descartes' contributions to the progress of mathematics and to the progress of philosophy are in fact connected.

If we turn to Descartes' own work for answers, his *Discourse on method* (1637) appears to offer an important clue. For here Descartes claims that his study of mathematics offered him a guide to understanding how we ought to approach the general problems of human knowledge, and he presents mathematics as offering the very standard for certainty that we ought to embrace when investigating what can be known in the domain of philosophy. Moreover, he urges us to produce 'long chains composed of very simple reasonings, [such as] geometers customarily use to arrive at their most difficult demonstrations' when we investigate 'all the things which can fall under human knowledge' (AT VI, pp. 18–19; CSM I, pp. 119–120).[2] Thus, the suggestion from Descartes himself is that a method characterized by deduction from simple, easily known objects serves as the thread that connects his mathematical work with his philosophical work.

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[1] Unless otherwise noted, citations from the *Geometry* are taken from Descartes (1954), and I use *G* to indicate references to this translation by Smith and Latham.

[2] In line with now standard citation format for Descartes' works, I use 'AT' to refer to Descartes (1996), 'CSMK' to refer to Descartes (1991), 'CSM' to refer to Descartes (1985), and 'WO' to refer to Descartes (1998).

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Using the suggestive remarks of the *Discourse* as a springboard, recent scholars have pursued a method-oriented strategy in their attempts to pinpoint a meaningful connection between Descartes' work in mathematics and philosophy. Put briefly, their goal has been to reveal similarities between the Cartesian method of mathematics and the Cartesian method of philosophy. For instance, Hintikka (1978) claims that Descartes' early philosophical works, as well as his later metaphysical writings, such as the *Meditations* (1641), show Descartes using a method to which he was exposed during the course of his mathematical studies. In particular, according to Hintikka, we find Descartes employing a modified version of the ancient method of analysis, which was prominent in the Platonist tradition of mathematics (Hintikka, 1978, p. 74).[3] A similar approach to the purported connection between Descartes' mathematics and philosophy is taken by Grosholz (1991), who agrees with Hintikka that there is a Cartesian method (in the singular) that binds Descartes' work in these domains together. She draws attention to the method Descartes employs in his mathematical work and attempts to build a bridge between the method used in the *Geometry* and the method that is applied to metaphysical problems in the *Meditations*, as well as in the *Principles of philosophy* (1644).[4] To be sure, the method-centered approaches taken by those such as Hintikka and Grosholz have gone a long way to illuminate some important similarities between Descartes' mathematical and philosophical work. But nonetheless, such approaches, which rely on Descartes' *Discourse* account of method, only inadequately capture the novelty of Descartes' approach to mathematical problem-solving.[5]

Careful attention to his innovations in geometry makes it difficult, if not impossible, I think, to maintain that the method Descartes actually employs from 1619 to 1637 is the same sort of deductive method that he describes in the *Discourse*. Though in the domain of geometry he lays focus on intuitively clear objects, namely, 'simple' curves, Descartes plainly ventures beyond a straightforward deductive method of reasoning in his programmatic treatment of these geometrically simple curves. For his goal in his early mathematical writings as well as in the *Geometry* is to distinguish legitimately 'geometric' curves from 'imaginary' non-geometrical curves, and to do so he lays emphasis on the clear and distinct *motions of construction* whereby curves are generated. This peculiar feature of Descartes' approach to mathematics guided his ground-breaking innovations in mathematics, and this feature resists straightforward subsumption under the sort of deductive method of reasoning he promotes in the *Discourse*. Thus, to take the sort of method-centered approach to Descartes' mathematics that Hintikka and Grosholz do is to miss a crucial moment in Descartes' work in mathematics, and his thinking about geometrical curves in particular. As a consequence, a method-centered approach leaves us a limited perspective on the connections that may in fact bind Descartes' mathematics with his philosophy.

To make better sense of how Descartes' peculiar innovations in mathematics may be connected to his philosophical program, I take the central role Descartes grants motion and construction in his study of geometrical curves as my point of departure, and aim to reveal a particular connection between Descartes' mathematics and philosophy that has gone unappreciated by those who have adopted a narrow view of his mathematical method. Specifically, I hope to show that Descartes attempts to incorporate his geometrical account of intelligibility into his philosophical work after 1628, and, in particular, as he composes his first metaphysics of nature, as presented in *Le monde* (1633). For in both the *Geometry* and *Le monde* intelligibility and exactness are conspicuously tied to clear and distinct motions for construction—in one case the construction of geometrical curves, and in the other God's construction of natural motions and the natural world more generally. Taking seriously the similar accounts of intelligibility and epistemological exactness assumed in these works, we see Descartes appealing to the very mathematical principles of construction that characterize his study of geometrical curves as he details his account of God's creation of the world. Moreover, there is, I think, a connection running in the other direction: the intelligibility of God's creation in *Le monde* allows Descartes to justify, at least implicitly, a contentious mathematical claim that he needs to situate his general solution to the Pappus problem in the program of the *Geometry*. In this manner, Descartes' metaphysics is fruitfully brought to bear on his mathematics.

Before turning to the specific ties between the *Geometry* and *Le monde*, I offer in Section 2 a brief overview of Descartes' early mathematical works, which illuminates the concerns with intelligibility and construction that remain constant throughout his mathematical researches. Appealing to the work of Bos (1981, 2001), I emphasize that, in the domain of geometry, the simple motions needed for the construction of legitimately 'geometric' curves remain the cornerstone of the standard of geometrical intelligibility that Descartes adopts throughout the most productive period of his mathematical work, 1619 to 1637. In Section 3, I detail Descartes' general solution to the Pappus problem, which he presents in Book I of the *Geometry*, and bring to light a tension in his argument for the 'geometrical' status of those curves that represent solutions to the Pappus problem. I then turn, in Section 4, to God's geometrical construction of nature in *Le monde*, and draw out connections between Descartes' early metaphysics and the mathematical program of his 1637 *Geometry*. As indicated above, my goal is to shed light on how Descartes brings geometrical intelligibility to bear on God's construction of nature and also on how the metaphysics of God's creation can help us address the problems plaguing Descartes' treatment of the Pappus problem.

2. Mathematics and philosophy from 1619 to 1628

In his early mathematical practice, Descartes followed in the footsteps of the ancients and investigated the methods by which to construct curves that could be used to solve geometrical prob-

[3] The account of ancient analysis that Hintikka (1978) invokes is explicitly drawn from Hintikka & Remes (1974).

[4] Grosholz is certainly not the only commentator to pay attention to the method of the *Geometry*. As she notes in her Introduction, others such as Vuillemin (1960) and Beck (1952) have treated the method of the *Geometry* in great detail in their attempts to situate Descartes' mathematics in the context of his philosophical corpus. I focus on Grosholz's account, both here and below, because her treatment of the *Geometry* nicely brings to light some problems we face if we too narrowly focus on method as that which connects Descartes' mathematics and philosophy. I should also note here that Grosholz's even more specific goal in Grosholz (1991) is to show that we find in Descartes' mathematical and philosophical works a common method of 'reductionism' and 'intuitionism': one that afforded Descartes a considerable amount of success in geometry and metaphysics but, ultimately, one that, she argues, prevented Descartes from advancing further than he actually did (ibid., pp. 2–5). I will return to some of her specific criticisms of Descartes' use of 'reductionism' and 'intuitionism' in geometry below.

[5] There is a further problem that plagues such approaches, which has been brought to light in Garber (1992). As Garber points out, it is difficult to maintain, as those such as Hintikka and Grosholz do, that issues of method remain central to Descartes' philosophical work after the *Discourse*. For in his post-1637 works, method is very rarely mentioned, and it is not explicitly granted a central role by Descartes in the metaphysics of the *Meditations* or the metaphysics and physics of the *Principles* (see Garber, 1992, pp. 48 ff.). See also Garber (1989) on this issue.

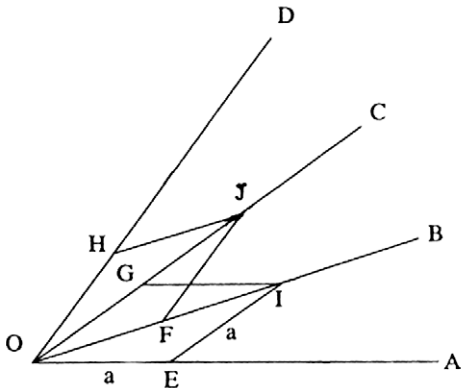

Fig. 1. The trisecting instrument (ca. 1619; adapted from Bos, 2001, p. 238).

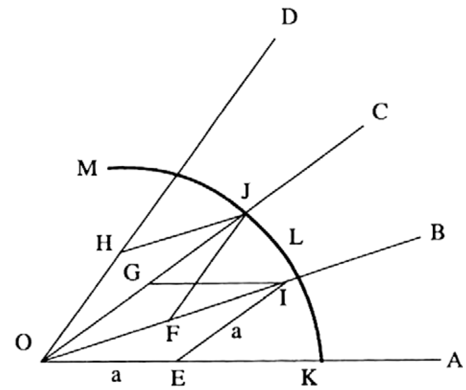

Fig. 2. Generating the curve KLM for trisection of an Angle (from Bos, 2001, p. 238; used with kind permission of Springer Science + Business Media).

lems. For instance, when Descartes tackled the classic problem of trisecting an angle, he contrived an instrument by which to generate a curve that could be used to trisect any given angle. We start with four rulers (OA, OB, OC, OD) that are hinged at point O (Figure 1). We then take four rods (HJ, FJ, GI, EI), which are of equal length *a*, and attach them to the arms of the instrument such that they are a distance *a* from O and are pair-wise hinged at points J and I. Leaving OA stationary, we now move OD so as to vary the measure of angle DOA from 0 to 180, and following the path of point J, we generate the curve KLM (Figure 2). As Descartes has it, we can construct the curve KLM on *any* given angle by appeal to the instrument described above, and then, by means of some basic constructions with straight lines and circles, the given angle can be trisected.[6] In this respect, the curve KLM is, for Descartes, the means for solving a geometrical problem, or, to be more specific, for solving a general class of problems, for the above mechanism and the resulting curve KLM can be used to trisect any given angle.

Though ancient geometers also constructed curves to solve geometrical problems, Descartes' use of an instrumental construction shows him advancing beyond the ancient standards of mathematical practice. According to ancient geometers (and Pappus in particular), the instrumental construction of a curve was considered *un*acceptable for geometrical problem-solving, because it allegedly lacked the same exactness as constructions that relied on straight lines and circles.[7] Descartes disagreed. He claimed that the constructions made with his 'new compasses' (such as the one seen above) are 'just as exact and geometrical as those drawn with ordinary compasses', because his instruments involve a single motion (Descartes to Beeckman, 26 March 1619; AT X, pp. 157–158; CSMK, pp. 2–3). In the case above, the single motion of OD is adequate for generating the triseting curve KLM, and this single motion is one which, according to Descartes, we can clearly conceive.

Constructions were *inexact* by Descartes' standards if they required 'distinct independent motions', which could not be clearly conceived. A favorite example of Descartes' is the spiral, whose construction requires the rectilinear motion of a point (P) and

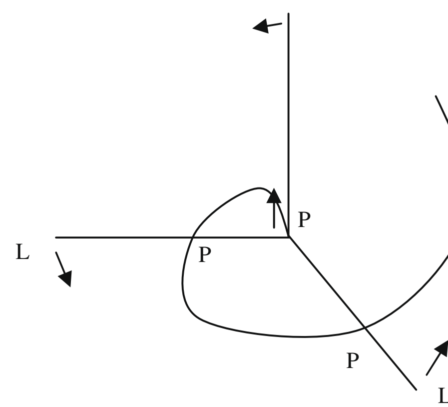

Fig. 3. Constructing a spiral.

the circular motion of a line segment (L) (Figure 3). We begin with point P at one end of the segment L, and then allow P to move uniformly toward the other endpoint as L moves uniformly in a circle. Following the path of P, we generate the spiral. As Descartes would explain in the 1637 *Geometry*, the problem with this sort of construction is that it relies on *simultaneous* rectilinear and circular motions. Descartes maintains that 'the ratios between straight and curved lines are not known', and 'cannot be discovered by human minds'; therefore, he concludes, 'no conclusion based upon such ratios [or relations] can be accepted as rigorous and exact' (*G*, p. 91). Thus, these sorts of constructions are, for him, inadmissible in the rigorous and exact domain of geometry, and as early as 1619 he deemed the curves generated by means of such constructions 'imaginary', which is for him equivalent to saying that they are non-geometrical. His new instrumental constructions, on the other hand, relied solely on the ratio (or relation) between straight lines, a ratio that is intelligible to us and which therefore meets the standards of geometrical exactness.[8]

6 To get a better sense of why this instrument can be used to generate a curve that can be used for trisection, notice that as OD moves (Figure 1), the angles DOC, COB, and BOA will remain equal in measure, regardless of where OD is positioned, owing to the way that the instrument is constructed. Namely, the rods HJ, FJ, GI, and EI are and will remain of equal length no matter where we position the arm OD, and this in turn preserves the congruence of the three angles DOC, COB, and BOA. KLM is thus the curve generated by a series of angles that have been trisected. My treatment of the trisecting instrument follows Bos' presentation in Bos (2001), pp. 237–239. See Bos (2001), p. 239, for the procedure Descartes uses to trisect a given angle once KLM has been constructed on the angle.

7 This was the letter but not the spirit of the law, for Pappus himself presents instrumental solutions in the *Mathematical collections*, the very same work in which the prohibition against instrumental constructions is made. For translations of Pappus' classification of curves and his apparent dismissal of instrumental constructions, see Cuomo (2000, p. 151) and Bos (2001), p. 37.

8 Descartes' claim regarding the unintelligibility of simultaneous straight- and curved-line motions is a centerpiece of his project in the *Geometry* to demarcate 'geometric' from 'imaginary' curves. Historically, however, it did not hold sway for very long. Very soon after the publication of the *Geometry*, Fermat and others had discovered methods for rectifying curves—that is, for determining the length of curved segments—which thereby challenged Descartes' claim of unintelligibility and, along with it, his program for distinguishing 'geometric' and 'imaginary' curves. See Bos (1981), pp. 314–315.

As we will see below, Descartes would later expand the domain of acceptable construction procedures beyond straight lines, circles, and his 'new compasses' as his studies progressed, but his concern with demarcating legitimately 'geometric' curves from non-geometrical curves remained a constant throughout his mathematical researches.[9] In 1619 he reports to Beeckman (in the letter dated 26 March) that he aims to show that some problems can be solved by straight lines and circles, some by appeal to his new compasses, and others only by use of non-geometrical 'imaginary' curves that lack exactness. If he is successful in demonstrating 'what sorts of problems can be solved exclusively in this or that way', then he is optimistic that 'almost nothing in geometry will remain to be discovered'.[10] He is, in this early period, also optimistic that he will be able to complete this 'gigantic task', and complete it he does, when the *Geometry* is published almost twenty years later (how successful his proposals are is an issue I will broach later in the paper). And as in 1619, Descartes will in 1637 appeal to the simple and intelligible motions involved in construction as the standard by which to distinguish 'geometric' curves from 'imaginary' ones.[11]

Before we take a closer look at the *Geometry*, it is important to note that the emphasis on clearly conceivable construction procedures that is characteristic of Descartes' early mathematical work is incorporated into the *Rules for the direction of the mind*, an incomplete philosophical work that Descartes began in 1619 and wrote in stages until its abandonment in 1628. Put briefly, his goal in this work is to develop a general science of all human wisdom, and he explicitly appeals to the method of mathematics as his guide (cf. Rule 4, especially AT X, pp. 377–378; CSM I, p. 19). Thus, we see that just as Descartes focused on simple, intuitively clear objects in his early geometrical researches, he indicates in the *Rules* that, in *all* our rational investigations, 'We must concentrate our mind's eye totally upon the most insignificant and easiest of matters, and dwell on them long enough to acquire the habit of intuiting the truth clearly and distinctly' (AT X, p. 400; CSM I, p. 33). A similar emphasis on cognition through the imagination (or the 'mind's eye') is apparent in Rules 13 through 21, where he addresses the methods appropriate for mathematical problem-solving. For instance, in Rule 14 he says that the 'perfectly understood' problems of mathematics 'should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect' (AT X, p. 438; CSM I, p. 56).

In the context of the *Rules*, the imagination clearly plays a foundational role for Descartes' new science of human wisdom, as well as for his general account of mathematical problem-solving. And as Schuster (1980) and Bos (2001) point out, this emphasis on the mental clarity afforded by the imagination fits well with what Descartes had achieved during the early stages of his mathematical research (ca. 1619–1620), when he was working with his 'new compasses' and had embraced the clearly and distinctly perceivable motions for constructing curves (by means of straight lines, circles, and well-defined instruments) as the standard for intelligibility and geometrical exactness. But as his mathematical research continued, Descartes became more comfortable using *algebraic* techniques to solve geometrical problems. For instance, jumping forward approximately six years to 1625–1626, Descartes investigated the method by which to determine the roots of any curve that can be represented by a 3rd- or 4th-degree equation, and discovered a general technique for solution.[12] The technique required that the curve's equation first be reduced to standard form: $x^4 = \pm px^2 \pm qx \pm r$.[13] The algebraic representation in turn indicates the type of construction that ought to be used for the solution, for any curve whose corresponding equation can be reduced to the above standard form requires a parabola and circle for the construction of its roots.[14]

As Descartes was composing the *Rules*, he tried to incorporate the new algebraic features of his mathematical method into his philosophical account of mathematical problem-solving. This is most apparent when we look at Rules 19 to 21, which were composed in the mid to late 1620s and which ended the work when he abandoned it in 1628 (AT X, pp. 468–469; CSM I, p. 76). In these rules, he tells us that we should reduce mathematical problems to equations, but he fails to cash out how exactly this is to be done; he simply presents these rules and offers no exposition. What is more problematic is that, in Rule 18, Descartes attempts to link geometrical construction with algebraic operations such as addition, multiplication, and division (AT X, pp. 461–468; CSM I, pp. 71–76), but the argument for a geometrical representation of these operations is ultimately unsuccessful.[15]

So while it is clear that from 1619 to 1628 mathematics serves as Descartes' model for rational thinking, and while it is also clear that Descartes tried to incorporate his new algebraic techniques into his general method for all rational sciences, he ultimately failed to build a bridge between his early mathematics and his early philosophy. In particular, he could not find a way to marry the algebraic operations he was integrating into his mathematical method with the construction of curves, or, more generally, with the cognition and movements of the imagination that were central to his early philosophy. As both Bos and Schuster have suggested, it was this failure that motivated Descartes' abandonment of the *Rules* in 1628, and Bos claims as well that, at this moment of abandonment, we see 'the beginning of the gradual separation of the

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9 In the 1637 *Geometry*, Descartes will again appeal to the instrumental construction of curves, but he is more reserved about admitting these construction techniques into the domain of geometry. As Bos points out, the focus of the *Geometry* is the construction of curves by means of ideal geometrical curves, and the instruments we find in the *Geometry* are meant to illustrate that composite motions can be used to generate clearly and distinctly conceivable curves. They are not meant to serve as legitimately geometrical construction procedures (Bos, 1981, pp. 309–310).

10 The full passage from the letter reads:

> I am hoping to demonstrate what sorts of problems can be solved exclusively in this or that way, so that almost nothing in geometry will remain to be discovered. This is of course a gigantic task, and one hardly suitable for one person; indeed it is an incredibly ambitious project. But through the confusing darkness of this science I have caught a glimpse of some sort of light, and with the aid of this I think I shall be able to dispel even the thickest obscurities . . . (To Isaac Beeckman, 26 March 1619; AT X, pp. 156–158; CSMK, p. 3)

Bos offers a slightly different translation in Bos (2001), p. 232.

11 In the text of the *Geometry*, Descartes refers to 'imaginary' curves, such as the spiral and quadratrix, as 'mechanical' curves, and claims to be modifying the ancient distinction between 'geometrical' and 'mechanical' curves. See Molland (1976) for Descartes' use of the ancient distinction to underscore the novelty of his program in the *Geometry*. I retain use of the term 'imaginary' in reference to the *Geometry* to make clear the continuity in Descartes' thinking from his early mathematical studies to 1637. Throughout the paper I also use 'geometric' and 'imaginary'—placed in single quotes—when I am referring to Descartes' peculiar interpretation of these terms.

12 The exact date of Descartes' work on 3rd- and 4th-degree equations is unknown, though we do know it must have occurred before 1628, the year he reported his solution to Beeckman. I follow Bos in dating the work to a time around 1625.

13 In the 1620s, it was already known that 4th-degree equations could be reduced to 3rd-degree equations. The method was discovered by Lodovico Ferrari in the sixteenth century, and later reprinted by Vieté early in the seventeenth century.

14 I will forgo the details of the solution here and refer the reader to Bos (2001), pp. 256–257.

15 See Schuster (1980), pp. 77–79; Bos (2001), pp. 266–270.

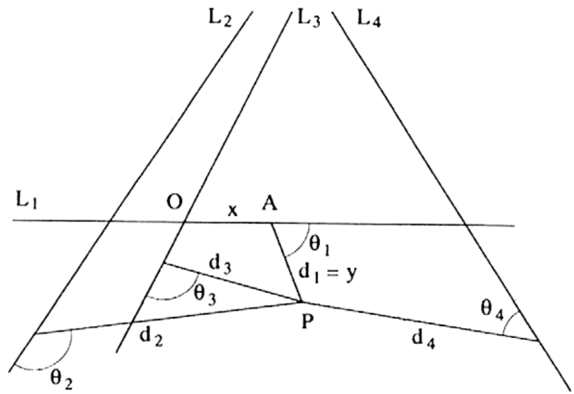

Fig. 4. The Pappus problem (from Bos, 2001, Fig. 19.1, p. 273; used with kind permission of Springer Science + Business Media).

ways of Descartes the mathematician and Descartes the philosopher' (Bos, 2001, p. 270). Certainly, mathematical reasoning continues to play a role in Descartes' later writings but, as Bos points out, the analogies that Descartes makes with mathematical method are less strict, even as early as his 1637 *Discourse*. On this reading, then, by 1628, the peculiar *innovations* that characterize Descartes' mathematical work cease to offer him a guide as he develops his mature philosophical program.

Though I agree with the general spirit of Bos' claim, his remark about the relationship between Descartes the mathematician and Descartes the philosopher deserves some further qualification. It is indeed the case that the *method* of mathematics is no longer explicitly linked with the *method* of philosophy in Descartes' later work as it is in the 1619-to-1628 period. However, there is a different strategy that I think Descartes adopts as he tries to bridge his mathematical work with his philosophical work in the post-1628 period. Namely, what I will emphasize below is that, in the 1632-to-1637 period, Descartes' conception of *intelligible motions* replaces his conception of method as that which binds his mathematics with his philosophy, and his metaphysics in particular. To appreciate the central role of motion in Descartes' post-1628 program, we need to turn our attention to his treatment of the Pappus problem, which he initiated in 1632 and developed more fully in 1637, when he published the *Geometry*. Attending to the tension in his proposed solution will open the way for a clearer understanding of how the early metaphysics of *Le monde* is connected to the mathematics of the *Geometry*, as well as a clearer understanding of the central role that intelligibility and motion play in these two seemingly unconnected works. Ultimately, I hope to show in the sections that follow that a standard of intelligibility grounded on simple and clearly conceivable motions is the thread that binds Descartes' multi-faceted 'rationalist' program together in this early period.

3. Descartes' solution to the Pappus problem[16]

Though around since the time of Euclid, the version of the Pappus problem that most concerned seventeenth century mathematicians was that presented by Pappus in Book VII of his *Mathematical collections*.[17] In brief, the problem is as follows (see Figure 4):

> *Given:* n lines L_i in the plane, n angles θ_i, a ratio β, and a line segment a. For any point P in the plane, let d be the oblique distance between P and L_i such that P creates θ_i with L.
> *Problem:* Find the locus of points P such that the following ratios are equal to the given ratio β:

For three lines:	$(d_1)^2$	:	d_2d_3
For four lines:	d_1d_2	:	d_3d_4
For five lines:	$d_1d_2d_3$	:	ad_4d_5
For six lines:	$d_1d_2d_3$	:	$d_4d_5d_6$

In general,

For an even $2k$ number of lines:			
	$d_{1\ldots}d_k$	:	$d_{k+1\ldots}d_{2k}$
For an uneven $2k + 1$ lines:			
	$d_{1\ldots}d_{k+1}$	:	$ad_{k+2\ldots}d_{2k+1}$

For any n-line Pappus problem, there are an infinite number of points P that satisfy the sought-after ratios. Following Bos, I call the locus of points P that represents the solution to a given n-line Pappus problem the 'Pappus curve'.

Descartes first tackled this problem in early 1632 at the urging of the Dutch mathematician Jacob van Golius. Unfortunately, it is difficult to know exactly what Descartes had discovered about the problem's solution at this point, because Descartes' initial reply to Golius is lost. Based on remarks in a follow-up letter from January 1632, we do know what Descartes *claimed* to have discovered about the general solution, namely, that the locus that is the solution for any Pappus problem can be traced by 'one single continuous motion completely determined by a number of simple relations', and these 'simple relations' are simple insofar as they 'involve only one geometrical proportion' (Bos, 2001, p. 350).[18] Descartes also differentiates Pappus curves from 'imaginary' curves, such as the spiral and the quadratrix (both of which are mentioned explicitly in the letter), based on his proposal that Pappus curves can be generated by continuous motions. The claims are not without their problems. First, as Bos points out, the terms 'single continuous motion', 'simple relations', and 'geometrical proportions' are vague, and Descartes' usage of these terms can be interpreted in different ways (ibid.). Second, as Bos also points out, it is difficult to believe that Descartes had at this time actually found a general method for tracing Pappus curves by continuous motion, not only because no such method is published by Descartes in the 1637 *Geometry*, where he treats the problem in great detail, but also because a

[16] My presentation of the general Pappus problem follows Bos (2001), pp. 272–273. Though there are different ways of presenting the problem, the version I use is the most helpful for making sense of Descartes' general solution to the problem, as presented in Book I of the *Geometry*. For an alternative account of the problem that is linked to the finding of tangents, see Boyer (1968), pp. 159–160.

[17] Mathematicians prior to Pappus attempted solutions to the problem, though, as he notes, they did not solve the general problem. For instance, when Apollonius proposed his solution, he used a technique that relied on his theory of conic sections and the transformation of areas. By means of this technique, also known as the application of areas, Apollonius could only provide solutions for Pappus problems of six lines or less, not the general problem.

[18] Bos (2001) provides a partial translation of the letter as well as the original text (p. 350).

general method was not published by any mathematician until the late nineteenth century.[19]

However, given the tools available to Descartes in 1632, it seems safe to say that, at the time he was writing to Golius, Descartes had determined that, at least in special cases of the Pappus problem,[20] the sought-after ratio could be expressed by an equation.[21] This is a claim that will be made in the 1637 *Geometry*, and, as we will see, appeal to the algebraic representation of the sought-after ratio plays a central role in Descartes' treatment of the problem in that context, where his goal is not simply to offer his general solution but also to show that all Pappus curves are 'geometric', that is, are constructible by legitimately clear and distinct motions, and are thereby fundamentally different from 'imaginary' non-geometrical curves.

In the *Geometry*, Descartes first treats the Pappus problem at the end of Book I. In this context he shows that, for any *n*-line Pappus problem, the sought-after ratio can be expressed as an equation. Based on the degree of the resultant equation, Descartes claims we can determine which curves are required to construct the locus of points P that serve as the solution to the problem. For instance, if the Pappus problem involves three, four, or five lines, but not five parallel lines, the equation that represents the ratio will be of degree 1 or 2 and we will use a ruler and compass to construct the locus of points. In this specific case, the Pappus problem is a 'plane' problem, for the solution is constructible by appeal to the intersection of the straight line and circle.

Descartes takes a different approach to the Pappus problem in Book II of the *Geometry*. Whereas in Book I Descartes investigates the curves necessary to construct the solution to the problem, in Book II he examines the form of the Pappus curve, that is, the form of the curve produced by the points P that satisfy the sought-after ratio. He again appeals to the equation that represents the sought-after ratio, but the classification is different. In this context, he tells us, for instance, that given a three- or four-line Pappus problem, the equation of the *Pappus curve* will have degree less than or equal to 2, and thus, the locus of points will be a curve of the first class: namely, a straight line, parabola, hyperbola, circle, or ellipse.[22] The concern here is not the types of curves required to construct a solution but rather the curve insofar as it is itself the solution to the problem.

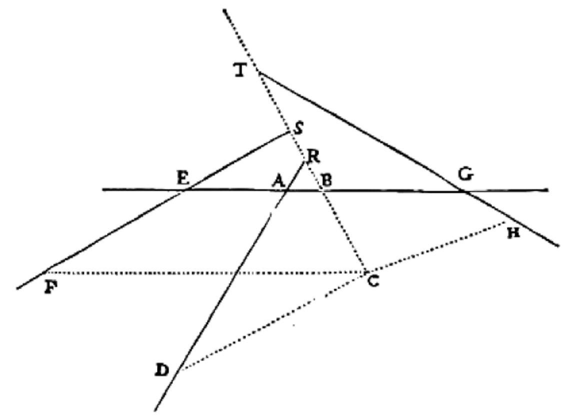

Fig. 5. The four-line Pappus problem in Book I of the *Geometry* (from *G*, p. 27; used with kind permission of Dover Publications, Inc.).

There is something curious about the placement of Descartes' account of the Pappus problem in the larger context of the *Geometry*. He claims in Book II that all 'geometric' curves can be constructed by clearly and distinctly perceived continuous motions (*G*, p. 43),[23] and moreover, that all 'geometric' curves can be represented by an equation (*G*, p. 48).[24] He also indicates that a general solution to the Pappus problem requires we both locate the locus of points P and trace the curve described by this locus (*G*, p. 22).[25] So, if it is the case that all Pappus curves are legitimately 'geometric', it then follows that these curves can be generated by suitably clear continuous motions. This, however, is not something that Descartes proves; he does not offer a general method of tracing Pappus curves by continuous motions to justify that the Pappus curves are legitimately 'geometric'. And perhaps even more curious is that in some cases Descartes explicitly appeals to the *point-wise* construction of Pappus curves, not constructions that rely on continuous motions.

For instance, at the end of Book I, Descartes presents his general solution to the Pappus problem and begins by reducing the sought-after ratio to an equation. Appealing to Figure 5, where our goal is to find all points C that meet the requirements of the problem, he considers the segments AB and BC as coordinates, and designates

[19] In particular, A. B. Kempe published a method for the continuous motion construction of any curve represented by an *n*th-degree equation. See Bos (1981), p. 324 n. 26.

[20] Such as in the above figure where we do not have *n* or *n*−1 parallel lines.

[21] Bos speculates that Descartes discovered in 1632 that the sought-after ratio could be expressed by an equation of the following form:

$$y(\alpha_2 x + \beta_2 y + \gamma_2)\ .\ .\ . = \delta(a)(\alpha_l x + \beta_l y + \gamma_l)(\alpha_{l+1} x + \beta_{l+1} y + \gamma_{l+1})\ .\ .\ .$$

where $l = k + 1$ if there are $2k$ lines, and $l = k + 2$ if there are $2k + 1$ lines.

The factor *a* on the right-hand side only occurs if the number of lines is uneven; δ is the given constant value of the ratio (Bos, 2001, p. 274). On Bos' account, Descartes' work on the Pappus problem in 1632 suggested the following to him as he continued his work in geometry:

(1) that curves should be accepted in geometry insofar as they were traced by geometrically legitimate motions;
(2) that these legitimately traced curves were precisely the Pappus curves;
(3) that Pappus curves were precisely the ones that admitted polynomial equations; and
(4) that therefore the totality of geometrically acceptable curves could be classified equivalently by the complexity of the tracing motion, the degree of the equation, and the number of given lines in the pertaining Pappus problem. (Ibid., p. 282)

In short, Bos suggests that Descartes' work on the Pappus problem around 1632 helped Descartes bring focus to the program of geometry that he would later present in 1637.

[22] Descartes' general classification of geometrical curves, which is based on the degree of their corresponding equations, is presented in Book II (*G*, p. 48). The first class of curves includes the circle, parabola, hyperbola, and ellipse: curves whose corresponding equations do not have degree higher than 2.

[23] '. . . if we think of geometry as the science which furnishes a general knowledge of the measurement of all bodies, then we have no more right to exclude the more complex curves than the simpler ones, provided they can be conceived of as described by a continuous motion or by several successive motions, each motion being completely determined by those which precede; for in this way an exact knowledge of the magnitude of each is always attainable.' (*G*, p. 43)

As noted above (n. 9), Descartes presents his account as an alternative to the ancient classification of 'geometrical' and 'mechanical' curves.

[24] 'I could give here several other ways of tracing and conceiving a series of curved lines, each curve more complex than any preceding one, but I think the best way to group together all such curves and then classify them in order, is by recognizing the fact that all points of those curves which we may call 'geometric', that is, those which admit of precise and exact measurement, must bear a definite relation to all points of a straight line, and that this relation must be expressed by means of a single equation.' (*G*, p. 48)

[25] 'Then, since there is always an infinite number of different points satisfying these requirements [of the Pappus problem], it is also required to discover and trace the curve containing all such points' (*G*, p. 22).

AB as y and BC as x. Since we know the measures of angles CRA and ABR from what is given in the problem, Descartes appeals to the properties of similar triangles to show that we can express the distance from C to the given lines by an equation in the two unknowns, y and x. In particular, referring to the case presented in Figure 5, he determines the equations that express the distances CD, CF, CB, and CH, where each equation includes only the two unknowns (*G*, pp. 29–30).[26] And now that we have algebraic representations of the distances between C and the given lines, what remains is that we find the specific points C such that d_1d_2 is to d_3d_4 as the given ratio β is to 1, which in the four-line Pappus problem above is equivalent to finding the points C such that CD^*CF is to CB^*CH as β is to 1. Each side of the ratio will have two unknowns, and after we multiply the distances, the equations on each side of the proportion will not have an unknown variable of degree higher than 2. Finding the values x and y that satisfy the ratio is now straightforward; what we must do is assign some value to x or y and then solve for the other variable. Here is how Descartes puts it:

> Assigning a value to y, we have $x^2 = \pm ax \pm b^2$, and therefore x can be found with ruler and compasses by a method [for constructing the roots of a curve represented by a 2nd-degree equation] already explained [earlier in Book I]. If then we should take successively an infinite number of different values for the line y, we should obtain an infinite number of values for the line x, and therefore an infinity of different points, such as C, by means of which the required curve could be drawn. (*G*, p. 34)

At first glance, there is nothing blatantly odd about Descartes' claim here, since it is in fact that case that we can substitute in values for y to determine the corresponding value for x, and thereby determine the locus of points with coordinates (x, y) that satisfy the requirements of the Pappus problem. What is odd, though, is Descartes' appeal to a *point-wise* construction for the Pappus curve; using the technique he describes, we generate the sought-after curve by locating points along the curve and then connecting the dots, so to speak.[27] But this is not the type of construction that we ought to be using for a curve that is legitimately 'geometric'; as he presents it, we should use a construction by *continuous* motion.[28] However, Descartes doesn't have a general method for tracing Pappus curves by continuous motion in his arsenal (and as noted above, no such method was published until the nineteenth century).

In the absence of a general method for tracing Pappus curves, Descartes instead tries to establish the status of Pappus curves as legitimately 'geometric' by exploiting the difference between the point-wise construction of Pappus curves and the point-wise construction of 'imaginary' curves. After presenting the point-wise construction of a five-line Pappus curve in Book II, he writes:

> It is worthy of note that there is a great difference between this method in which the [Pappus] curve is traced by finding several points upon it, and that used for the spiral and similar curves. In the latter, not any point of the required curve may be found at pleasure, but only such points as can be determined by a process simpler than that required for the composition of the curve . . . On the other hand, there is no point on these curves which supplies a solution for the proposed problem that cannot be determined by the method I have given. (*G*, pp. 88–91)

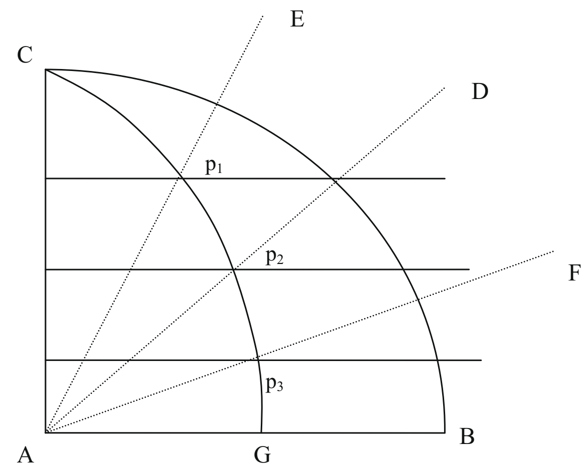

Fig. 6. The point-wise construction of the quadratrix CG.

To understand the different methods of construction Descartes is drawing attention to here, consider the point-wise construction of the quadratrix, a curve Descartes considers 'imaginary'. We begin with the arc CB, and by means of straightedge and compass, we divide the arc into equal parts (Figure 6). We then divide the radius AC into equally many equal parts; in Figure 6, we have divided both the arc (and therefore the angle CAB) and the segment CA into four equal parts. We now draw horizontals extending from the points of division along CA, which will intersect the arc BC, and locate points p_1, p_2, and p_3 where the horizontals intersect the segments dividing the angle CAB. Connecting these points we generate the quadratrix CG.

What Descartes emphasizes in the above passage (though not by appeal to this or any specific example) is that we cannot find arbitrary points along the curve when we use a point-wise construction of an 'imaginary' curve. In the case of the quadratrix, for instance, we are only able to divide the given arc into 2^n parts given the restrictions of Euclidean construction; namely, we bisect the original angle by straightedge and compass, bisect the two resulting angles, and so on. As such, it is not possible to divide the arc any way we please, and we cannot therefore locate any arbitrary point along the curve by use of point-wise construction. In the case of the Pappus curves, however, we can find any arbitrary point on the curve by appeal to the equations corresponding to the problem; borrowing Bos' terminology, Descartes is claiming that Pappus curves can be generated by 'generic' point-wise constructions.

Having thereby distinguished the point-wise construction of Pappus curves from the point-wise construction of 'imaginary' curves, Descartes makes a further and very contentious assertion: the 'generic' point-wise constructions we can use to generate Pappus curves are equivalent to constructions that rely on continuous motions. He states, in particular, 'this method of tracing a curve by determining a number of its points taken at random applies only to curves that can be generated by a regular and continuous motion' (*G*, p. 91). Here Descartes assumes, without argument, that if we

[26] Based on his treatment of the four-line problem above, Descartes posits that this same method can be used regardless of the number of lines with which we begin (see *G*, p. 33).

[27] A similar allowance for point-wise constructions of Pappus curves is found in Book II (*G*, p. 335). See Bos (1981), p. 316, for discussion of the Book II example.

[28] Actually, given what Descartes presents in Book I, there is no possibility of using a construction by continuous motion. He tells us that we should use straight lines and circles in our construction, yet the curve we are constructing is a conic section (as he tells us in Book II), and to construct a conic section by the continuous motion we would need to use conic sections in our construction. Thus, the only way to construct the Pappus curve and follow Descartes' criteria for construction is to use the sort of point-wise construction that Descartes describes, whereby we determine different points along the Pappus curve by substituting values into the given equation. See Bos (1981), pp. 302–303.

can find arbitrary points along a curve using a point-wise construction, then we could also trace the curve by continuous motion. He does not, however, prove this equivalency, and if we are to try to find an argument, it seems the best we can do is assume, with Descartes, that the distinction he proposes between the point-wise construction of Pappus curves and the point-wise construction of 'imaginary' non-geometric curves renders the Pappus curves non-imaginary and therefore 'geometric' in his sense.

To make this problem clearer, we can schematize the argument for the 'geometric' status of Pappus curves as follows:

(1) For any *n*-line Pappus problem, we can reduce the problem to an equation.
(2) Using the equation, we can arbitrarily determine points on the locus by substituting values into the equation (which we cannot do in the case of 'imaginary' non-geometric curves such as the quadratrix).
(3) If we can arbitrarily determine points on the locus by substituting values into the equation, then the locus could also be constructed by continuous motions.
(4) If the locus can be constructed by continuous motions, then the locus is a 'geometric' curve.
(5) Therefore, any Pappus curve is a legitimately 'geometric' curve.

It is, of course, claim (3) that is problematic. Descartes asserts this equivalency between 'generic' point-wise constructions and constructions by continuous motions without proof, and even without much argument. Thus, what Descartes needs to establish he merely assumes, namely, that there is a general method for tracing Pappus curves by continuous motion.

This tension in Descartes' presentation of the Pappus problem is the focus of Bos (1981), where Bos uses this example to highlight the difficulties Descartes faced as he attempted to bring algebraic techniques to bear on geometrical problem solving. Later, Grosholz (1991) would claim that Descartes is forced to make his contentious equivalence between 'generic' point-wise construction and construction by continuous motions because of the 'reductionist' and 'intuitionist' approach he takes in the *Geometry*. In particular, his attempt to reduce the foundations of geometry to intuitively clear simple motions and simple objects leaves him no other option; his method and his chosen foundations for geometrical reasoning simply prevent adequate treatment of more complicated curves. And this is precisely what Grosholz claims Descartes' approach to the Pappus curves reveals: a 'conceptual poverty of his starting-points' (Grosholz, 1991, p. 33). Had Descartes broadened his outlook, he would not have missed the import of 'his abstract relational structures'—the equations he uses to represent curves—and could have, according to Grosholz, advanced further in his geometrical researches (ibid., p. 50).

While Grosholz is correct that Descartes fails to provide a deductively valid argument for the equivalence between 'generic' point-wise constructions and constructions by continuous motions, to suggest that he failed to offer a *legitimate justification* for this claim rests on the assumption that we should hold Descartes to the standard of the particular methodology she outlines, namely, a standard encapsulated in a 'reductionist' method of mathematics. Even though in the *Discourse* Descartes promotes the use of a method that begins with simple, clear, and distinct objects and then builds clear and distinct chains of reasoning from this foundation (AT VI, 18–20; CSM I, 119–120), it is not altogether clear that this is the proper standard by which to evaluate the merits of Descartes' strategy in the *Geometry*.[29] On the one hand, as Bos has convincingly argued and as outlined above, Descartes conceived of geometry as 'the science of solving geometrical problems by the construction of points through the intersection of curves' (Bos 1981, p. 331), 'not . . . as an axiomatic, deductively ordered corpus of knowledge about points, lines, *etc.*' (ibid., p. 327). To therefore hold Descartes to the standards of an axiomatic, deductive system, as Grosholz seems to do, flies in the face of the very program he offers in the *Geometry*. On the other hand, and building on Bos' characterization of Descartes' program of geometry, Grosholz's assessment does not pay due attention to the central role of construction in the *Geometry*, and, in particular, to the relationship Descartes forges between clear and distinct motions for construction and his accepted standard of geometrical intelligibility. While she does admit, albeit implicitly, that intelligibility is wedded to the intuitively simple objects and simple motions that Descartes adopts as his starting-point for investigation,[30] she does not consider the possibility that this focus on intelligible motions is meant to *replace* a model of intelligibility centered on method—which for Grosholz is the other horn of Descartes' dilemma in the *Geometry* as well as his later philosophical works.

By taking seriously the relationship Descartes forges between construction, motion, and intelligibility in the *Geometry*, I think we can make better sense of the controversial equivalence Descartes makes between point-wise and continuous motion constructions by turning to *Le monde*, a metaphysical work in which construction, motion, and intelligibility also play a central role, this time as Descartes outlines God's creation of nature. As I will argue in the following section, the creation story presented in *Le monde* offers an indication of why Descartes may have taken the equivalency between 'generic' point-wise constructions and constructions by continuous motions to be intelligible and therefore acceptable in the domain of *Geometry*, even without a deductively valid mathematical proof for this equivalence at hand.

4. God's creation in *Le monde*

Le monde was written between October 1629 and 1633, and includes two major sections: *Treatise on light* and *Treatise on man*. In the *Treatise on light*, Descartes offers his account of a 'new world' that is intended to serve as a more convincing and intelligible model than that offered by the Scholastics. In short, Descartes is attempting to replace their 'old', earth-centered world of forms and qualities with a 'new', Copernican, sun-centered world of matter in motion.[31]

In presenting his new world, Descartes does not make a direct argument for his mechanical model of nature. Instead, his presentation is hypothetical, and he uses a fable that details God's creation of the world and through which he hopes the truth of his claims will be revealed. The standard for what is admissible in his creation story is intelligibility, a standard that he claims distinguishes his account from the unintelligible Scholastic account of nature. He writes:

> my purpose, unlike theirs, is not to explain the things that are in fact in the actual world, but only to make up [*feindre*] as I please a world in which there is nothing that the dullest minds cannot conceive, and which nevertheless could not [in reality] be cre-

[29] See Grosholz (1991), pp. 5–6, for her explicit appeal to the sections of the *Discourse* referenced above as the basis for her analysis of Descartes' method in the *Geometry*.

[30] On this score, Grosholz remarks:

> 'I do think that trying to reduce curves to points and lines, or to ideal instruments that can construct them is philosophically misguided, since inquiry into what makes a curve a curve, which is the propaedeutic for generating interesting problems about curves, must first pay attention to its peculiar integrity.' (Grosholz, 1991, p. 50)

[31] As a historical note, Descartes suppressed *Le monde* in its entirety in November 1633 after hearing of Galileo's condemnation, which occurred in June of that same year.

> ated exactly the way I have imagined [*feint*] it. (AT XI, p. 36; WO, p. 24)

While Descartes admits that basing his account on the standard of intelligibility forces him to relinquish the absolute truth of his fable, it nonetheless places the creation story on firm ground as a *possible* way in which God created the world and grants it more plausibility than the less intelligible world of the Scholastics. As Descartes puts it:

> since everything I propose here can be imagined [*imaginer*] distinctly, it is certain that even if there were nothing of this sort in the old world [of the Scholastics], God can nevertheless create it in a new one; for it is certain that He can create everything we imagine [*imaginer*]. (Ibid.)[32]

The hypothetical account of creation Descartes offers in Chapters 6 and 7 of *Treatise on light* runs as follows:

(1) God chooses a finite area of infinite space and creates matter in it (Chapter 6).
(2) Upon creating matter, God also imposes motion on each part; that is, God endows each part of matter a particular direction and a particular speed (Chapter 6).
(3) The speed and direction granted to the parts of matter result in the formation of material objects as we experience them and explain all the changes we witness in nature (Chapter 6).
(4) Since God is immutable, He conserves the motion of matter in the same way He created it (Chapter 7).
(5) There are rules (or laws) of motion that govern material bodies and that ultimately rest on God's immutability and unchanging conservation of natural motions (Chapter 7).

The central role that Descartes grants matter and motion in this new world is apparent: the world begins as formless matter and takes on the forms we see in nature because God grants each part of matter a particular motion. The motion that Descartes appeals to, however, is *not* the obscure motion of the Scholastics, but is modeled instead on the motions embraced by the geometers. He writes:

> the nature of the motion that I mean to speak of here is so easily known that even geometers, who among all men are the most concerned to conceive [*concevoir*] the things they study very distinctly, have judged it simpler and more intelligible than the nature of surfaces and lines, as is shown by the fact that they explain 'line' as the motion of a point and 'surface' as the motion of a line. (AT XI, p. 39; WO, p. 26)

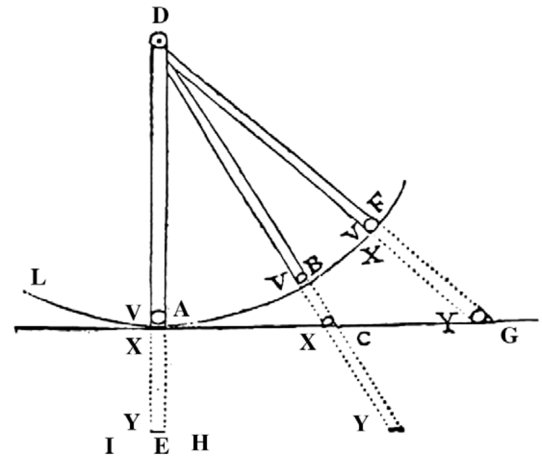

Fig. 7. The sling in Rule 3 (from WO, p. 31; used with kind permission of Cambridge University Press).

While he mentions no specific geometer by name here, it is at least clear that Descartes himself should be included in the group of geometers to which he appeals.[33] For as we saw above, Descartes was quite comfortable appealing to the motions of points and lines to construct curves, both in the *Geometry* and his earlier mathematical works. And here, in the context of *Le monde*, the same sort of geometrical motions employed by Descartes are now ascribed to God's creation of matter in the new world that Descartes describes, because these motions are more intelligible and 'easily known' than the motion characteristic of Scholastic metaphysics.[34]

There is also a further and deeper connection we can draw between Descartes' account of motion in *Le monde* and the motions of construction that characterize his mathematical work. For recall that in his geometrical work, Descartes appeals to the construction of curves by *continuous* motions in order to demarcate legitimately 'geometric' curves from 'imaginary' non-geometrical curves. This very same restriction on legitimately 'geometric' and thereby intelligible curves is transferred to the intelligible motions of Descartes' new world, as we see when he remarks in *Le monde*, 'I know of no motion . . . which is easier to conceive [*concevoir*] of than the lines of the geometers, by which bodies pass from one place to another and *successively occupy* all the spaces in between' (AT XI, p. 40; WO, p. 27; my emphasis). The visible motions of his new world are thereby modeled on the continuous motions used by geometers to generate clearly conceivable curves; and thus, we see Descartes integrating his approach to geometry, and his geometrical standard of intelligibility in particular, with his early mechanical account of nature.

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[32] I note the French term of what Gaukroger translates as 'imagine' in this and the previous passage to highlight the different terms Descartes uses in these contexts. In the previous passage (AT XI, p. 36; WO, p. 24), Descartes uses the term *feindre*, which can be more literally translated as 'to feign' or 'to make up'. In the passage above, however, Descartes invokes *imaginer*, which has a closer affinity to 'to conceive' than *feindre*, and which at the least is not meant to indicate the activity of making something up.

[33] In his editorial note on the passage cited above, Gaugroker points out that we see here Descartes putting his own interpretive gloss on the role of motion in geometry, for the above 'accounts/definitions in terms of motions are not to be found in the three best-known geometers—Euclid, Archimedes, and Apollonius' (WO, p. 26 n. 53). While Gaukroger is correct to say that Euclid defines line as breadthless length (*Elements*, Book I, Definition 2) and surface as that which has length and breadth only (*Elements*, Book I, Definition 5), some ancient geometers did describe the construction of curves by appeal to the motion of points. For instance, Apollonius describes the construction of the spiral by appeal to the simultaneous motions of a line and point in a manner similar to that used by Descartes:

> 'If a straight line is drawn on a plane and if after having rotated uniformly round one of its extremities which is held fixed, it returns whence it started, and during the rotation of this line a point is carried uniformly with respect to itself through the length of the line, beginning at the extremity that is held fixed, this point will describe a spiral in the plane.' (*Spirals*, in *Opera*, 2:50–52, cited in DeGandt, 1995, p. 203)

Moreover, the passage from Descartes reveals his embrace of a kinematical understanding of geometry, which was prevalent in the seventeenth century. See DeGandt (1995), pp. 202–209, on the integration of motion into geometry by sixteenth- and seventeenth-century mathematicians such as Galileo, Toricelli, and Roberval.

[34] Of the Scholastic account of motion, Descartes writes:

> 'They themselves admit that the nature of their motion is very little understood. And trying to make it more intelligible, they have still not been able to explain it more clearly than in these terms: *Motus est actus entis in potentia, prout in potentia est*. These terms are so obscure to me that I am compelled to leave them in Latin because I cannot interpret them.' (AT XI, 39; WO, p. 26)

Further examination of *Le monde* reveals yet another connection between Descartes' mathematics and metaphysics, one which will help us better understand his treatment of the Pappus problem in the *Geometry*. Consider in particular the third of the three rules that govern the continuous motions in his new world:

> [Rule 3: W]hen a body is moving, even if its motion most often takes place along a curved line . . . nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is the inclination they have to move, is different from their motion. (AT XI, pp. 43–44; WO, p. 29)

To get a better handle on what Descartes is proposing in Rule 3, consider his example of the motion of a ball in a sling (Figure 7). We observe the ball moving along an arc from L to F, but if we consider the ball's motion at an instant, we find that it has a centrifugal tendency in a straight line. For instance, when the ball is between points V and A, it has a straight line tendency, or inclination, toward E; that is, the ball *would* continue along the rectilinear path to E if it were not constrained by the sling. The same holds for all the points through which the ball moves as it traces the path from L to F: when the ball is between points V and B, it has a tendency toward Y, when between V and F a tendency toward G, and so on. Thus, the ball's visible motion along the arc LF is analyzable in terms of the ball's tendency to motion at each point it occupies along the curve.

What Descartes' analysis of the ball reveals is that all the continuous motions we witness in nature can be reduced to a series of motions at an instant, namely, instantaneous motions that result from the composition of natural straight line tendencies and the 'unnatural' disposition of the matter surrounding the moving object.[35] On this account, then, apparently continuous motions are fundamentally discontinuous, and as Descartes clarifies, it is God who is ultimately responsible for a body's motion at every instant along its path:

> This rule rests on the same foundation as the other two, and depends solely on God's conserving everything by a continuous action, and consequently on His conserving it not as it may have been some time earlier but precisely as it is at the very instant He conserves it. So, of all motions, only motion in a straight line is entirely simple and has a nature which may be grasped [*comprise*] wholly in an instant. For in order to conceive [*concevoir*] of such motion it is enough to think that a body is in the process of motion in a certain direction, and that this is the case at each determinable instant during the time it is moving. (AT XI, pp. 44–45; WO, pp. 29–30)

Notice the connection that Descartes draws here between God's simple action and the motions he deems conceivable at an instant. Ultimately, according to the creation story Descartes presents, the straight line tendencies of bodies in motion derive from God's continual and immutable conservation of the motion of all parts of matter, where in general, God imposes direction and speed onto every part of matter. However, at any given instant, all we can conceive is the direction of God's push, so to speak; and because of the limits of what we can humanly conceive, this direction at an instant must be in a straight line. Put differently, God creates motion in the simplest possible way, where simplicity is determined by appeal to that which is clearest and most distinct to the human intellect—the very same standard embraced in the geometrical works written around the same time.[36]

I want to suggest that here, in the domain of metaphysics—where Descartes appeals to intelligibility as his standard for describing God's creation of the world—we find a justification for the contentious equivalence that Descartes presented in the *Geometry*. For recall that in order to maintain the status of Pappus curves as geometrically intelligible, Descartes had to assume that curves generated by 'generic' point-wise constructions were also constructible by continuous motions. In the context of the *Geometry*, there is no mathematical argument presented to support the equivalency, but looking at what Descartes presents in *Le monde*, we find that curves generated by continuous motions are in fact reducible to instantaneous motions, or, more precisely, to the infinite points of motion along the curve. In the case of the ball in the sling, it traces a continuous path along the curve from L to F; that is, there is no break in its visible motion. But as Descartes points out, its continuous motion can be understood as an infinite series of pushes, where, in this metaphysical context, it is God who imposes the instantaneous straight line pushes, which ultimately explain the body's motion.[37]

Though neither God's activity nor any metaphysical claim plays an explicit role in Descartes' program of geometry, I want to suggest that this metaphysical account of continuous motions in nature presented in Rule 3 provided Descartes with a model for understanding the equivalency of 'generic' point-wise constructions and constructions by continuous motion. For since in both the domain of metaphysics and the domain of mathematics the standard of admissibility is the same—namely, *intelligibility* renders curves and motions acceptable—to say that God's point-wise construction of continuous curves in nature is acceptable in metaphysics is to say at the same time that any such 'generic' point-wise construction—one according to which *every* point along the curve has a determinable value—could serve as the explanation for the generation of continuous curves, which is precisely the controversial claim that Descartes makes in the domain of geometry.[38] There is of course an important disanalogy between the two cases insofar as mathematical points are not attributed tendencies or forces. But if

[35] See AT XI, pp. 46–47; WO, p. 30, where Descartes says that 'it is the various dispositions of matter that render the motions irregular and curves'.

[36] See Schuster (1977), Ch. 8, for an account of this Rule, which emphasizes the limits Descartes places on God's activity but does not emphasize the criterion of intelligibility that I claim underwrites Descartes' claims.

[37] As Descartes puts it:

> 'According to this rule [Rule 3], then, we must say that *God alone is the author of all the motions in the world in so far as they exist* and in so far as they are straight, but that it is the various dispositions of matter that render the motions irregular and curved.' (AT XI, pp. 46–47; WO, p. 30; my emphasis)

In the remainder of this passage, Descartes draws a connection between God's actions, straight lines, and moral values:

> 'Likewise, the theologians teach us that God is the author of all our actions, in so far as they exist and in so far as they have some goodness, but that it is the various dispositions of our wills that can render them evil.' (Ibid.)

Based on these remarks, we can generate the following ratio: God's activity : irregularity :: straight : curved :: goodness : evil.

[38] In light of my interpretation, notice that Descartes can preserve the important distinction between the point-wise construction of 'geometric' curves and the point-wise construction of 'imaginary' curves that was so crucial to his classification of curves. For given that God's activity applies to *every* point along the continuous curves found in nature—given, that is, that God's construction is a 'generic' point-wise construction—the model of construction and intelligibility in *Le monde* does not support the intelligibility of 'imaginary' curves. For recall that, according to Descartes, there are points along 'imaginary' curves, such as the quadratrix, for which values cannot be determined. In other words, these curves cannot be 'generically' point-wise constructed (cf. the discussion appended to Figure 6, above). My thanks to Matthew Holtzman and Marco Panza for urging me to clarify this point.

we consider the geometrical case as an idealization of the motions of bodies, then the absence of a force or a tendency in a particular direction does not render the reduction of continuous motion to motions at an instant any less acceptable in the domain of geometry, where we generate continuous curves by appeal to point-wise constructions.

So while the equivalency of 'generic' point-wise constructions to constructions by continuous motions is presented in the *Geometry* without an explicit argument, my suggestion is that it is *not*, as Grosholz suggests, merely presented as an ad hoc assumption that will allow Descartes to maintain his intuitionist-reductionist program of philosophy, or even maintain his geometrical program for that matter. With the standard of intelligibility as the common thread running through the metaphysics of *Le monde* and the mathematical program of the *Geometry*, Descartes could, at least implicitly, rely on the intelligibility of God's creation of natural motions to sustain the intelligibility of 'generic' point-wise constructions in mathematics and thus its acceptability in his program of geometry. Without a general method for tracing all Pappus curves at his disposal, this, it seems, is actually the best that Descartes could do with the resources available to him.

5. Conclusion

On the account I have presented above, the main thread tying Descartes' mathematics and metaphysics together during his early career is an account of intelligibility grounded on clear and distinct motions for construction. It is this standard of intelligibility that we see at play in the *Geometry* as Descartes attempts to demarcate legitimately 'geometric' curves from 'imaginary' non-geometrical curves and which we also see at play in *Le monde* as Descartes describes God's creation of matter. It is also by appeal to this standard of intelligibility that we get a better sense of how Descartes can justify his contentious claim in the *Geometry* that all curves constructible by 'generic' point-wise constructions are also constructible by continuous motions, for as we see in Descartes' exposition of Rule 3 of *Le monde*, the continuous motions of nature are reducible to the simple, straight-line pushes that God imposes on matter at each instant. Taking seriously the intelligibility of simple motions in the mathematics and metaphysics that Descartes develops during this period of his career thus grants us a view of the connection between his mathematics and philosophy that is sensitive to the innovations of Descartes' early mathematics and, in this sense, grants us deeper insight into the connection between Descartes' mathematics and philosophy than an approach that assumes a common mathematical and philosophical method as the thread binding together Descartes' work in these domains.

Looking forward to the post-1637 period, paying due attention to the standard of intelligibility that Descartes invokes in his different domains of inquiry can, I think, shed further light on Descartes' struggle to connect mathematics, metaphysics, and natural philosophy in his mature works. Though I cannot fill in the details of such an account here, I will suggest that Descartes no longer adopts a standard of intelligibility wedded to simple motions in either the *Meditations* or *Principles*. For in neither context is there reference to those construction procedures that rendered geometrical curves intelligible in Descartes' early mathematical work; his emphasis instead is on the clarity and distinctness of ideas, whether the idea of God or the ideas of mathematical figures, which are presented to the mind already constructed, so to speak.[39] What this suggests is that Descartes had to refashion his understanding of mathematical knowledge as he attempted to integrate mathematical certainty with the metaphysics of the *Meditations* and as he attempted to integrate the mathematical features of material bodies with the physics of the *Principles*.[40] While the details will be left for a later time, I hope that I have at least made a convincing case that such a transition in Descartes' thinking about mathematics and mathematical intelligibility is lost if we focus too heavily on the methods of Descartes' mathematical and philosophical work, and it goes unappreciated unless we pay due attention to the standards of intelligibility that thread his work together during the different stages of his mathematical and philosophical career.

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[39] See for instance Descartes' characterization of the idea of the triangle in the Fifth Meditation:

> 'When, for example, I imagine a triangle, even if perhaps no such figure exists, or has ever existed, anywhere outside my thought, there is still a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented by me or dependent on my mind.' (AT VII, p. 64; CSM II, pp. 44–45)

[40] As apparently confirmed by Descartes' report to Mersenne, a year after the *Geometry* was published, that he would like to devote more time to a 'sort of geometry where the problems have to do with the explanation of natural phenomena' (Letter to Mersenne, 27 July 1638; AT II, p. 268; CSMK, pp. 118–119). Descartes' changing views of mathematics are briefly discussed in Garber (2000).

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